Relationship between scanning resolution and viewing angle on view sphere with perspective

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Abstract: This article provides relationship between scanning resolution of acquisition device and flare of angle of viewing cone. Knowing the technical parameters of equipment, a viewing representation adapted to this device can be generated. The representation of polyhedral object is obtained from viewing sphere with perspective model.

Keywords: Polyhedron view model, scanning resolution, view sphere

1 Introduction

Visual identification system generally works in two stages:

1. At the first stage in a system memory are stored representations of objects that then the system will be able to recognize
2. In the second stage takes place correct recognition of 3D objects in the scene.

Thus the selection of the adequate representation of objects and their generating is one of the key steps in the recognition system. Representation of 3D object can be divided into two categories: object-centered (where object is described in the related coordinate system) and viewer-centered (where object is described in the coordinate system connected to the observer). In viewer-centered representation aspect graph is widely used. The idea of aspect graph (conceptually very similar to the viewing representation) was proposed by Koenderink and van Doorn [2]. Arbel , and Ferrie [1] introduce a method for distinguishing between informative and uninformative viewpoints from views are obtained. In this paper we deal with the viewer-centered category. The class of objects to recognize we limit to the polyhedra. To represent this class of objects viewing representation of polyhedron is used. This representation can be obtained from the viewing sphere with perspective.

The concept of generating 3D viewing representation based on assumed generation space model can be described as follows:

- Circumscribe a sphere on a polyhedron. The sphere is small (radius *r*) and its center is at the polyhedron center.
- On this sphere place a space view cone *vc* with angle of flare 2α. The vertex of this cone is a model *viewing point VP*. Viewing axis always goes through sphere and the object center.
- Unconstrained movement of the cone vertex, where the cone is tangent to the small sphere creates a large sphere with radius *R*. This sphere is called *viewing sphere* (fig. 1.1). Each object has its own viewing sphere, the same for all views of this particular object.
- Generate views, taking into account only object features selected for identification i.e. faces of polyhedron. Faces visible in the viewing cone create *a view*, external edges from this view create *view contour*.

Complementary cone cc is a cone defined by current viewing axis (it's collinear with its height) and has an opposite direction of flare to the viewing cone. Angle of complementary cone equals $\pi - 2\alpha$.

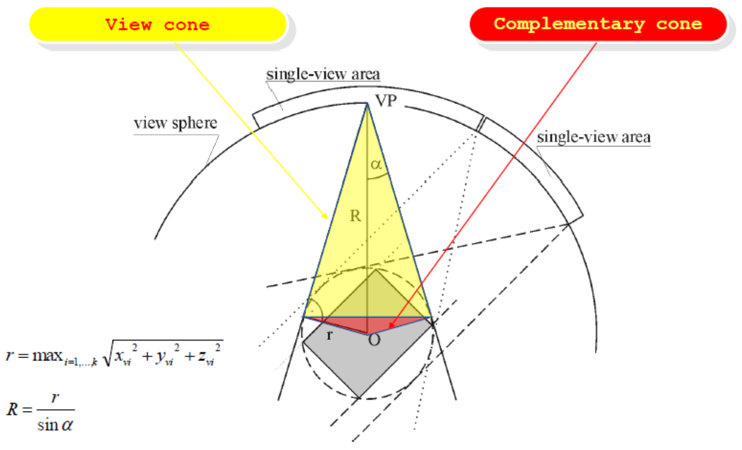

Figure 1.1. View sphere with view and complementary cone

View is created by faces that are visible in the viewing cone at a certain viewing point *VP* position. External edges of a view create a viewing contour.
On fig. 1.2 the view sphere divided into "one-view" areas is shown. A some view associated with region also are illustrated. One-view area is the area on view sphere from which the same view is obtained, i.e. the same set of distinguished features. As a distinguished feature we choose face because it is very characteristic to polyhedron and quite easy to recognize.

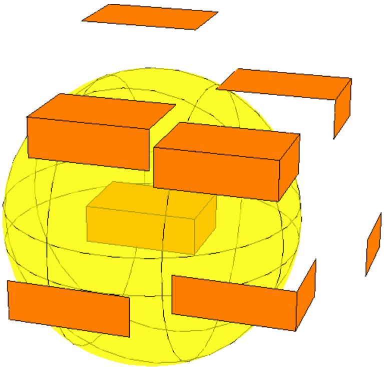

Figure 1.2. Some views of a cuboid and corresponding "one-view" areas on the view sphere

Viewing representation of polyhedron is a complete set of all possible views (for a certain angle). Completeness of representation means that all possible views of a polyhedron are obtained.

Methods of generating 3D viewing representation of polyhedron for object visual identification are described in several papers. In first group of methods called iterative, central views corresponding to object features chosen for identification are generated [3,4]. Then, single-view areas (corresponding to views generated earlier) are calculated on the view sphere. In each step the covering of the whole viewing sphere by with single-view areas is checked. This process is repeated in iterative way until we get a complete cover of view sphere.

Papers [6, 7] describes noniterative methods, which are better because they are faster. Complete representation is obtained by covering the viewing sphere precisely with single-view areas without loop, but in spiral way and controlling "edge" register (of not covered area). These methods have been designed to generate views of convex polyhedra. In some algorithms in this group the notion of complementary cone rotated around each face of polyhedron is used and and these methods may be applied to certain class of concave polyhedra. During this movement visual events in viewing points are registered [5].

2 Scanning resolution in spiral methods

2.1 Spiral methods of view representation generation

Yet another group of methods of generating view representation is spiral movement of view point on view sphere [8]. The idea follows from method when complementary cone were rotated over each face of polyhedron. In this method trajectory of movement of *VP* is spiral. In this way we scan the whole space over each single face of polyhedron (see fig 2.1a). Repeat this operation for each face, a complete set of views is obtained. The main disadvantage of this method are overlapping scan areas (see fig 2.1b).

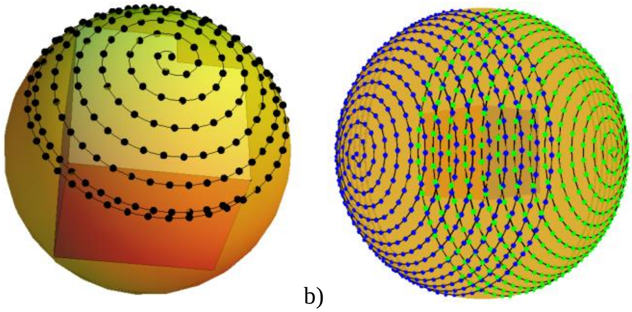

Figure 2.1. a) View points on spiral trajectory over one face of polyhedron
b) overlapped regions of scanning

Thus was established a method of scanning the whole sphere from one spiral path [10] (fig. 2.2).

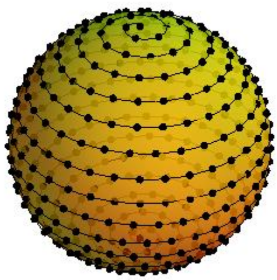

Figure 2.2. Viewing points on spiral trajectory of scanning on view sphere

In determining the formula describing the movement of viewing point following assumptions were made:

- whole sphere should be scanned in one path
- trajectory should be spiral path
- viewpoints should be distributed on a sphere as evenly as possible.

The formula for this movement is follows:

$$\begin{cases} x = R \cdot \cos(\frac{2\pi}{\sigma}t) \cdot \sin(t) \\ y = R \cdot \sin(\frac{2\pi}{\sigma}t) \cdot \sin(t), \\ z = \cos(t) \end{cases}$$

where σ denotes required resolution and t is non-linear parameter (to obtain uniform distribution on sphere) defined as follows:

$$\begin{cases} t = \arcsin(1-s); \text{for } s \in [0,1] \\ t = \pi - \arccos(t)\,; \text{for } s \in (1,2] \end{cases}.$$

The number of viewing points on view sphere for the given resolution is:

$$N = \frac{4\pi + \sqrt{2}\sigma}{2\sigma^2}.$$

Method for generating views in this method can be described in three main steps:

1. Determination of viewing points distributed on a spiral trajectory on view sphere for the given scan resolution.
2. Generation of view from every view point and add it to the collection of views.
3. Removing repeated views from a set of generated views.

Doing in this way a complete set of views is determined, so a view representation of polyhedron is obtained.
The advantage of this method is lack of overlapping scan areas (occurring in the method of spiral motion over faces), which leads to determining significantly smaller number of viewpoints, and a ordered set of viewpoints (convenience in construction of other structures, e.g. to generate aspect graphs).

2.2 Dependency between scanning and device resolution

With the growing popularity of scanners generating range map, view representation of object seems to be useful. Due to three dimensional structure of view polyhedron can be easily matched with range map. An example of such a method is described in [9].
The obvious question that is worth asking is what is the relationship between scanning resolution and resolution of device that collect data (like range scanner). According to the view sphere with perspective model resolution of device corresponds to viewing angle on view sphere, a view sphere is a place from device collect data.
On fig. 2.3 view sphere with radius R is shown. Recall that the small sphere is a sphere circumscribed on the polyhedron, r stands for radius of this sphere. Let us denote device resolution by σ_d and scanning resolution respectively by σ_s. L is a distance on small sphere between points on this sphere.

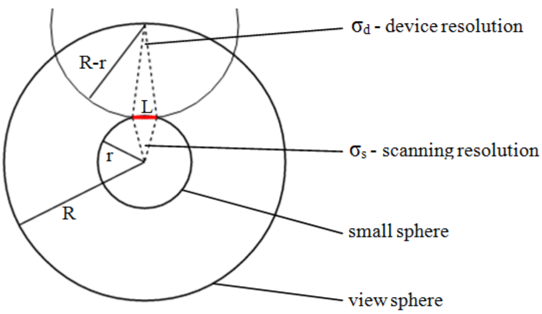

Figure 2.3. The relationship between the scanning resolution and the resolution of the device that obtain data

One can observe, that

$$L = r \cdot \sigma_s$$

and

$$L = (R - r) \cdot \sigma_d.$$

Hence, the relationship between scanning resolution (denoted by σ_s) and scanning resolution of device (denoted by σ_d) like range scanner can be described by the formula

$$\sigma_s = \frac{R - r}{r} \cdot \sigma_d.$$

In this equation the relationship between scanning resolution and resolution of acquisition device is demonstrated, R depends on flare of viewing cone.

This estimation is useful in tasks of this type: find the known polyhedral objects acquiring information from the scene using a range scanner with a certain resolution. When we know technical parameters of acquisitions device like scanning resolution, than we are ready to generate complete view representation of polyhedral objects. Completeness means that all possible views, with a given accuracy depending on resolution , are generated.

3 Results

Spiral algorithms of views generation were tested on a number of monotonous polyhedrons. On figure 3.1 an example of tested polyhedron monotonous polyhedron with 8 faces.

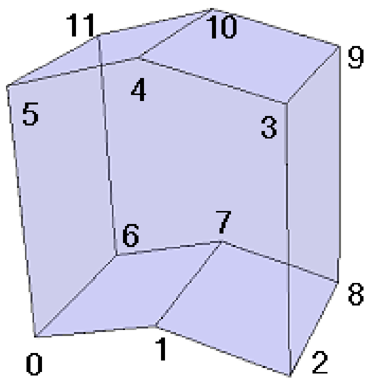

Figure 3.1. Tested polyhedron with 8 faces

Vertices coordinates and faces of the polyhedron defined by its vertices are show in table 3.1.

Table 3.1. Vertices and faces of tested polyhedron with 8 faces

Vertex index	Coordinates
0	(-4.0, -4.0, 3.0)
1	(0.0, -3.0, 3.0)
2	(4.0, -4.0, 3.0)
3	(4.0, 4.0, 3.0)
4	(0.0, 5.0, 3.0)
5	(-4.0, 4.0, 3.0)
6	(-4.0, -4.0, -3.0)
7	(0.0, -3.0, -3.0)
8	(4.0, -4.0, -3.0)
9	(4.0, 4.0, -3.0)
10	(0.0, 5.0, -3.0)
11	(-4.0, 4.0, -3.0)

Face index	Vertices indices
0	1, 0, 6, 7
1	2, 1, 7, 8
2	3, 2, 8, 9
3	4, 3, 9, 10
4	5, 4, 10, 11
5	0, 5, 11, 6
6	1, 2, 3, 4, 5, 0
7	6, 11, 10, 9, 8, 7

List of complete views of polyhedron shown on fig. 3.1 is presented in table 3.2.

Table 3.2. Views of tested polyhedron with 8 faces

Index	Faces in view
0	0 6
1	1 6
2	0 6
3	0 1 6
4	5 6
5	2 6
6	1 5 6
7	0 2 6
8	3 6
9	4 6
10	3 4 6
11	0 1 2 6
12	2 3 6
13	4 5 6
14	0 1 5 6
15	2 3 4 6

Index	Faces in view
16	3 4 5 6
17	0 1 2
18	0 2
19	2
20	2 3
21	2 3 4
22	3 4
23	3 4 5
24	4 5
25	5
26	1 5
27	0 1 5
28	0 1
29	3 4 7
30	3 4 5 7
31	4 5 7

Index	Faces in view
32	5 7
33	1 5 7
34	0 1 5 7
35	0 1 7
36	0 1 2 7
37	0 2 7
38	2 7
39	2 3 7
40	2 3 4 7
41	4 7
42	3 7
43	7
44	1 7
45	0 7

On fig. 3.2 an another tested polyhedron with 10 faces is shown. Each view is described by indices of faces which are visible in this view.

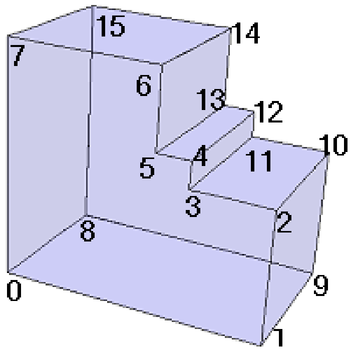

Figure 3.2. Tested polyhedron with 10 faces

This polyhedron is described by vertices coordinates and its faces (defined by indices of vertices), see table 3.3.

Table 3.3. Vertices and faces of tested polyhedron with 10 faces

Vertex index	Coordinates
0	(-5.0, -4.0, 3.0)
1	(5.0, -4.0, 3.0)
2	(5.0, 1.0, 3.0)
3	(2.0, 1.0, 3.0)
4	(2.0, 2.0, 3.0)
5	(0.75, 2.0, 3.0)
6	(0.75, 5.0, 3.0)
7	(-5.0, 5.0, 3.0)
8	(-5.0,-4.0,-3.0)
9	(5.0, -4.0, -3.0)
10	(5.0, 1.0, -3.0)
11	(2.0, 1.0, -3.0)
12	(2.0, 2.0, -3.0)
13	(0.75, 2.0, -3.0)
15	(0.75, 5.0, -3.0)
15	(-5.0, 5.0, -3.0)

Face index	Vertices indices
0	0,1,2,3,4,5,6,7
1	2,1,9,10
2	3,2,10,11
3	4,3,11,12
4	5,4,12,13
5	6,5,13,14
6	7,6,14,15
7	0,7,15,8
8	1,0,8,9
9	15,14,13,12,11,10,9

Table 3.4 presents obtained views of polyhedron with 10 faces shown on fig. 3.2. Due to generality of presented spiral method of view generations allows to use this

also for a much broader class of polyhedra. The challenge may be to develop the best possible method of storing their views to match with collected data from real world.

Table 3.4. Views of tested polyhedron with 10 faces

Index	Faces in view
0	0
1	0 5
2	0 2
3	0 2 5
4	0 2 4
5	0 3 5
6	0 2 4 5
7	0 2 3 5
8	0 2 3 4 5
9	0 8
10	0 5 8
11	0 3 5 8
12	0 1 3 5
13	0 2 4 6
14	0 7
15	0 2 4 5 6
16	0 2 7
17	0 1 2 3 5
18	0 2 4 7
19	0 1 2 3 4 5
20	0 2 3 4 5 6
21	0 7 8
22	0 1 3 5 8
23	0 2 4 6 7
24	0 1 2 3 4 5 6
25	2 3 4 5 6
26	2 4 5 6
27	2 4 6
28	2 4 6 7
29	2 4 7
30	2 7
31	7
32	7 8
33	8
34	5 8
35	3 5 8
36	1 3 5 8
37	1 3 5
38	1 2 3 5
39	1 2 3 4 5
40	1 2 3 4 5 6
41	1 3 5 8 9
42	1 3 5 9
43	1 2 3 5 9
44	1 2 3 4 5 9
45	1 2 3 4 5 6 9
46	2 3 4 5 6 9
47	2 4 5 6 9
48	2 4 6 9
49	2 4 6 7 9
50	2 4 7 9
51	2 7 9
52	7 9
53	7 8 9
54	8 9
55	5 8 9
56	3 5 8 9
57	2 3 4 5 9
58	2 4 9
59	9
60	3 5 9
61	2 4 5 9
62	2 9
63	2 3 5 9
64	5 9
65	2 5 9

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