# Many-Valued Gates for Reducing the Chip-Area of Integrated Circuits 

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#### Abstract

In this paper are proposed new many-valued gates $\boldsymbol{K}-\mathbf{P L A}, \boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ and $\boldsymbol{T}(\boldsymbol{K} / \mathbf{2})$ for a logical synthesis of digital integrated circuits. The semi-custom integrated circuit K-PLA has the architecture of a Programmable Logic Array of a type $\boldsymbol{A N D} \boldsymbol{D} \boldsymbol{O R}$ and includes new $\boldsymbol{K}$-valued valves MAX, MIN and $\operatorname{GATE}(\boldsymbol{A}, \boldsymbol{j})$. A gate $\boldsymbol{T}(\mathbf{2} / \boldsymbol{K})(\boldsymbol{T}(\boldsymbol{K} / \mathbf{2})$ ) is intended for transformation binary ( $\boldsymbol{K}$-valued ) entrance words into $\boldsymbol{K}$-valued (binary) output words. The method of the logical synthesis with the use $\boldsymbol{K}-\mathbf{P L A}, \boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ and $\boldsymbol{T}(\boldsymbol{K} / \mathbf{2})$ allows to reduce nearly three times the chip-area, which is essential for placing of the circuit's realization of the system of partial Boolean functions.

Keywords: Programmable Logic Array, logical synthesis, semi-custom integrated circuit, manyvalued gate, reducing of chip-area.


In this paper we look at the relevant problem of minimization of a chip-area of integrated circuits.

This paper is the obligatory supplement of our article [7], where are absent figures for explanation of functioning new many-valued gates.

The work done in this area includes the Dagon [1], MIS [2,3] and Ceres [4] systems. Results [5] and [6] were received with the help of the library basis of gates. For example, the paper [5] communicates about gate "AND/OR/NAND/NOR constraint".

Our method of the synthesis of circuits [7] allows to sinthesize integrated circuits with new many-valued gates $\boldsymbol{K}-\mathbf{P L A}, \boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ and $\boldsymbol{T}(\boldsymbol{K} / 2)$. The input information for synthesis is a system of partial Boolean functions ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ). For example, we sinthesize the system $(\mathbf{1 8 , 6 , 2 0})$, showed on Fig.1.

There are main stages of our method of logical synthesis of integrated circuits [7].
1.The choice of the quantity of $\boldsymbol{K}$ for gates in a circuit's realization of the system of partial Boolean functions ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ). We take that $\boldsymbol{K}$, where $\boldsymbol{\operatorname { l o g }}_{2} \boldsymbol{K}=\left[\boldsymbol{\operatorname { l o g }}_{2} \boldsymbol{K}\right]$.
2. The coding the binary input and output words of ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ) by $\boldsymbol{K}$-valued words with length $n=] m / \log _{2} K[$ and $s=] r / \log _{2} K[$.

In this stage we construct the system of partial many-valued logical functions ( $\boldsymbol{n}, \boldsymbol{s}, \boldsymbol{q}$ ), which corresponds to system ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ). For our system (18,6,20), showed on Fig. 1, we get the system (6,2,20), showed on Fig. 2.

| $\mathbf{x 1}$ | $\mathbf{x 1 8}$ | $\mathbf{g 1} \quad \mathbf{g 6}$ | v1 | v6 | f1 $\mathbf{f 2}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 100011000110011100 | 011100 | 430634 | 3 | 4 |  |  |
| 100010111100100101 | 110110 | 427445 | 6 | 6 |  |  |
| 111101100011010010 | 010011 | 754322 | 2 | 3 |  |  |
| 000100100011100111 | 000110 | 044347 | 0 | 6 |  |  |
| 101100101011000101 | 101101 | 545305 | 5 | 5 |  |  |
| 100011100110010011 | 010001 | 434623 | 2 | 1 |  |  |
| 000010110100111001 | 010101 | 026471 | 2 | 5 |  |  |
| 100110001101110000 | 101101 | 461560 | 5 | 5 |  |  |
| 000110100111001001 | 110110 | 064711 | 6 | 6 |  |  |
| 111110001000110101 | 111111 | 761065 | 7 | 7 |  |  |
| 100010110111010010 | 000101 | 426722 | 0 | 5 |  |  |
| 100011100110110100 | 011101 | 434664 | 3 | 5 |  |  |
| 000110001101001011 | 110100 | 061513 | 6 | 4 |  |  |
| 100011100001100111 | 000001 | 434147 | 0 | 1 |  |  |
| 111000110110000111 | 011011 | 706607 | 3 | 3 |  |  |
| 111011000110110100 | 111111 | 730664 | 7 | 7 |  |  |
| 100011100111001001 | 100100 | 434711 | 4 | 4 |  |  |
| 001001110010000111 | 111011 | 116207 | 7 | 3 |  |  |
| 100111001001000111 | 110001 | 471107 | 6 | 1 |  |  |
| 101100100111000110 | 011010 | 544706 | 3 | 2 |  |  |

Fig. 1.
Fig. 2.
Fig. 1. System of partial Boolean funtions $(\mathbf{1 8 , 6}, 20)$
Fig. 2. System $(6,2,20)$ of partial 8 -valued logical functions
3. The construction of the circuit $S$ with $T(2 / K), K-P L A$ and $T(K / 2)$ for realization ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ). The circuit $\boldsymbol{S}$ is showed on Fig. 3.


Fig. 3. Circuit $\boldsymbol{S}$ for synthesis of $(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})$
4. The minimization of the system of partial $\boldsymbol{K}$-valued logical functions $(\boldsymbol{n}, \boldsymbol{s}, \boldsymbol{q})$.

With that aim we generate system-intervals which realize by implicants and construct SOP-system in basis $\left\{\vee, \wedge, v_{i}^{j}, \mathbf{1 , 2}, \ldots, \boldsymbol{K}-\mathbf{1}\right\}$.
5. The creation the special matrixes for realization of the concrete parameters of gates for $\boldsymbol{K}$ - PLA with the help of the $\boldsymbol{K} \boldsymbol{- P L A}$ - programmer.
6. The completion of the synthesis with the help of the $\boldsymbol{K} \mathbf{- P L A}$ - programmer.

It is easy to explain structures of all blocs showed on Fig. 3 with the help of the concrete examples.

Fig. 4 shows the structure of translator $\boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ for three input pins and $\boldsymbol{K}=\mathbf{8}$, where $\qquad$ is the $\qquad$ gate NOT, $\triangle$ is the gate $A N D$,is the gate for realization of operation Max,is the gate $\operatorname{GATE}(\boldsymbol{A}, j)$.
The gate $\boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, \boldsymbol{j})$ has one input pin and one output pin, where $\boldsymbol{y}=\boldsymbol{A}$ for $\boldsymbol{x}=\boldsymbol{j}$ and $\boldsymbol{y}=\mathbf{0}$ for $\boldsymbol{x} \neq \boldsymbol{j}$. The main parameters $\boldsymbol{A}$ and $\boldsymbol{j}$ for $\boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, \boldsymbol{j})$ we must be able to modify with the help of a special equipment called by programmer (similar as PROM blower).

If input signals of $\boldsymbol{T}(\mathbf{2} / \mathrm{K})$ are equal to $\boldsymbol{x i 1}=1, x i 2=0, x i 3=1$, to input signal of $\operatorname{GATE}(5,1)$ is equal to 1 and it's output signal is equal to 5 . Then the output signal of $\boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ is equal to 5 also.


Fig. 4. Structure of translator $\boldsymbol{T}(\mathbf{2} / \boldsymbol{K})$ for three input pins

Fig. 5 shows the structure of translator $\boldsymbol{T}(\boldsymbol{K} / \mathbf{2})$ for one input pin, three output pins and $\boldsymbol{K}=\boldsymbol{8}$, where $\nabla$ is the gate $\boldsymbol{O R}, \square$ is the gate $\boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, \boldsymbol{j})$.

For example, if input signal of $\boldsymbol{T}(8 / 2)$ is equal to $f i=4$, input signal of the gate $\boldsymbol{\operatorname { G A T E }}(\mathbf{1 , 4})$ is equal to 4 so and output signal of the gate $\boldsymbol{\operatorname { G A T E }}(\mathbf{1 , 4 )}$ is equal to 1 . Then output signals of $\boldsymbol{T}(2 / K)$ are equal to $g i 1=1, g i 2=0, g i 3=0$.

Figure 6 shows the structure of $\boldsymbol{K} \boldsymbol{P} \boldsymbol{L A}(\mathbf{6 , 2 , 2 0})$ for $K=8$ with 6 input pins, 2 output pins and 20 intermediate lines. This block includes gates MIN, MAX, $\boldsymbol{\operatorname { G A T E }}(7, j), \boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, 7)$, which are described as $\boldsymbol{\Delta}, \boldsymbol{\nabla}$, and $\square$.


Fig. 5. Structure of translator $\boldsymbol{T}(8 / 2)$ with one input pin, three output pins

The parametr $\boldsymbol{A}$ of each gate $\boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, \boldsymbol{j})$ from $\boldsymbol{M}(\boldsymbol{A N D})$ is equal to $\boldsymbol{K} \mathbf{- 1}$ (in our case $\boldsymbol{A}=7$ ) and the parametrs $\boldsymbol{j}$ must be turn by the $\boldsymbol{K} \boldsymbol{-} \boldsymbol{P} \boldsymbol{L} \boldsymbol{A}$ - programmer with the help of the matrix for turning (Fig. 7, Fig. 9). The parametr $\boldsymbol{j}$ of each gate $\boldsymbol{\operatorname { G A T E }}(\boldsymbol{A}, \boldsymbol{j})$ from $\boldsymbol{M}(\boldsymbol{O R})$ is equal to $\boldsymbol{K}-\boldsymbol{1}$ (in our case $\boldsymbol{j}=7$ ) and the parametrs $\boldsymbol{A}$ must be turn by the K-PLA - programmer with the help of the matrix for turning (Fig. 7, Fig. 9).


Fig. 6. Structure of 8-PLA(6,2,20)

For effective synthesis of $\boldsymbol{8 - P L A}$ we can minimize the obtained $\boldsymbol{8}$ - valued logical functions (Fig. 2).

Using our tools [7], we have the results of the minimization - system's intervals for the system (6,2,20). The corresponding SOP-system is showed in [7].


Fig. 8. Structure of $\mathbf{8 - P L A}$ for realization $(\mathbf{6 , 2 , 2 1})$

SOP-system is easily to realize by $\boldsymbol{8} \mathbf{- P L A}(\mathbf{6}, 2,21)$. In our case we have the circuit, where the structure of $8-\operatorname{PLA}(6,2,21)$ is showed on Fig. 8 and the matrix for turning is showed on Fig. 9.

At last we have the circuit for realization our system of Boolean funtions $(18,6,20)$. Our circuit $S$ is showed on Fig. 10 .

Our method of synthesis [7] and new many-valued elements $\boldsymbol{T}(\mathbf{2} / \boldsymbol{K}), \boldsymbol{T}(\boldsymbol{K} / 2)$ and K-PLA allow considerably to reduce the chip-area, which is needed for realization the system of partial Boolean functions ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ).

We will estimate the effect (chip-area minimization) of the using of our method with $K=4$.

| v1 v6 | f1 f2 | v1 v6 | f1 f2 |
| :---: | :---: | :---: | :---: |
| 430634 | 34 | 488888 | 81 |
| 427445 | 66 | 884828 | 28 |
| 754322 | 23 | 888878 | 28 |
| 044347 | 06 | 888788 | 82 |
| 545305 | 55 | 888884 | 38 |
| 434623 | 21 | 788888 | 83 |
| 026471 | 25 | 886888 | 83 |
| 461560 | 55 | 888808 | 38 |
| 064711 | 66 | 880888 | 84 |
| $\boldsymbol{M}($ AND $)=761065$ | $\boldsymbol{M}(\mathbf{O R})=77$ | $\boldsymbol{M}(\boldsymbol{A N D})=888818$ | $\boldsymbol{M}(\mathbf{O R})=44$ |
| 426722 | 05 | 888885 | 55 |
| 434664 | 35 | 826888 | 85 |
| 061513 | 64 | 888868 | 85 |
| 434147 | 01 | 888808 | 58 |
| 706607 | 33 | 887888 | 66 |
| 730664 | 77 | 048888 | 86 |
| 434711 | 44 | 068888 | 68 |
| 116207 | 73 | 878888 | 68 |
| 471107 | 61 | 084888 | 86 |
| 544706 | 32 | 788868 | 77 |
|  |  | 188888 | 78 |

Fig. 7
Fig. 9

Fig. 7. Matrix for turning $8-\operatorname{PLA}(6,2,20)$ (without minimization)
Fig. 9. Matrix for turning $8-\operatorname{PLA}(6,2,21) \quad$ (with minimization)

The use of traditional 2-PLA $(\boldsymbol{m}, r, \boldsymbol{q})$ allows to realize any system of partial Boolean functions ( $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ ) by a circuit with $\boldsymbol{L}(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})$ commutation points between input/output pins and intermediate lines, where

$$
\begin{equation*}
L(m, r, q)=(2 m+r) * q . \tag{1}
\end{equation*}
$$

The use of our method of the synthesis with elements $\boldsymbol{T}(\mathbf{2 / 4}), \boldsymbol{T}(\mathbf{4} / \mathbf{2})$ and $\mathbf{4}$ $\boldsymbol{P L A}(\boldsymbol{m} / \mathbf{2}, \boldsymbol{r} / \mathbf{2}, \boldsymbol{q})$ allows to realize the same system of partial Boolean functions $(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})$ by an other circuit $\boldsymbol{S}$, where the chip-area is equal to

$$
\begin{equation*}
L_{s}(m, r, q)=\frac{m}{2}(20+q)+\frac{r}{2}(12+q) . \tag{2}
\end{equation*}
$$



Fig. 10. The circuit for realization $(\mathbf{1 8 , 6}, \mathbf{2 0})$

The formula (2) was obtained from [7] by the assumption: parameters of $\boldsymbol{m}$ and $\boldsymbol{r}$ are even numbers. This assumption limits not the domain of using our method, because parameters $\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q}$ of "industrial" PLA are even numbers always.

We define the effect as

$$
\begin{aligned}
& c(m, r, q)=L(m, r, q)-L_{s}(m, r, q)=(2 m+r)^{*} q- \\
& -\left(\frac{m}{2}(20+q)+\frac{r}{2}(12+q)\right)=m q+q(m+r)-\frac{q}{2}(m+r)^{-10 m-6 r=} \\
& =m q+\frac{q}{2}(m+r)-10 m-6 r=\frac{2 m q+m q+r q-20 m-12 r}{2}= \\
& =\frac{q(3 m+r)-12(3 m+r)+16 m}{2}=\frac{(3 m+r)^{*}(q-12)+16 m}{2} .
\end{aligned}
$$

Like that the reducing of the chip-area for the $\operatorname{2-PLA}(m, r, q)$ is equal to
$c(m, r, q)=\frac{(3 m+r)^{*}(q-12)+16 m}{2}$.
Now we can estimate the effect of the using of our method for a circuit's realization of any system of partial Boolean functions ( $M, \boldsymbol{R}, \boldsymbol{Q}$ ) with $\boldsymbol{M} \geq \boldsymbol{m}, \boldsymbol{R} \geq r, \boldsymbol{Q}$ $\geq q$.

It's known [ 8 ] (formula (4.19) ), what any ( $\mathbf{M}, \boldsymbol{R}, \boldsymbol{Q}$ ) may be realized by a circuit in the basis $\{\mathbf{2 - P L A}(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})\}$, where the number of elements $2-\operatorname{PLA}(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})$ is no more than
] $\frac{Q}{q}\left[(] \frac{M-m}{m-r}[+] \frac{R}{r}[)\right.$.
As the use of our method allows to replace a circuit $S$ with one $\mathbf{2 - P L A}(m, r, q)$ by the equivalent circuit $S$ with $\boldsymbol{T}(\mathbf{2} / 4), T(4 / 2)$ and $\mathbf{4 - P L A}(m / 2, r / 2, q)$ and to reduce the chip-area on $\boldsymbol{c}(\boldsymbol{m}, \boldsymbol{r}, \boldsymbol{q})$ we have the effect for any $(\boldsymbol{M}, \boldsymbol{R}, \boldsymbol{Q})$, which is equal to
$C(M, R, Q)=] \frac{Q}{q}\left[(] \frac{M-m}{m-r}[+] \frac{R}{r}[) \frac{(3 m+r)(q-12)+16 m}{2}\right.$

## Example

Let the circuit's realization of the system $(\mathbf{6 4 , 6 4 , 4 0 0 0})$ contains 560 (with the help of (4)) $\mathbf{2 - P L A}(\mathbf{1 6}, 8,100)$. Then we can replace this circuit by the equivalent circuit $S$ with $T(2 / 4), T(4 / 2)$ and $\mathbf{4 - P L A}(8,4,100)$, where the chip-area is smaller from (5) on $\mathbf{1 4 5 1 5 2 0}$ commutation points between input/output pins and intermediate lines.

For the realization of one $2-\operatorname{PLA}(\mathbf{1 6}, 8,100)$ is needed (1) the chip-area, which is equal to 4000.

Then we mark that the chip-area, which was economized by using of our method, allows to place 362 elements 2-PLA(16,8,100).

In that way our method allows to reduce nearly in three times the chip-area, which is essential for placing of the circuit's realization of the system $(\mathbf{6 4 , 6 4}, 4000)$ in comparison with the method [8].

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