Assessment of economic activity of the company on the base of fuzzy inference rules

Andrzej Chojnacki[1], Hanna Borzęcka[1]
[1] Military Uniwersity of Technology
Faculty of Cybernetics

Abstract. The article presents the assessment of companies efficiency, the idea based on the fuzzy set theory and fuzzy logic. Relevant financial measures are selected by experts and auditors. Variables of the proposed fuzzy model are expressed as linguistic variables and their correlations are defined by If-Then fuzzy rules of conclusion.
The theoretical modeling is then followed by numerical examples.
There is further extension of model toward assessment and comparison for enterprises of different size and various business branches.

Keywords. Fuzzy sets theory, fuzzy logic, fuzzy rule of conclusion, linguistic variables, fuzzy model of assessment.

1 The Word is not bivalent

The theory of fuzzy logic (theory worked out by L. A. Zadeh in 1965) and fuzzy inference system have many technical applications.

Problem with description of complex and non-precise concepts was the reason of development of the many-valued logic.

The answer for this problem was description in natural language. Natural language uses qualitative and imprecise terms. It is by nature not ambiguous and imprecise so description in natural language of classical logic and sets theory is imperfect and difficult. It causes necessity of description other then classical which bases on crisp intervals and dichotomous phenomena. Fuzzy logic was founded as an answer for this problem – many valued logic and fuzzy inference system. Three-valued logic and mathematic introduced by Polish logician Jan Lukasiewicz was the first attempt replacement of two-valued logic of Aristoteles.

Fuzzy logic has many applications because reality is in its nature is not crisp, not univocal and can not be precisely defined.

Fuzzy logic as well as theory of fuzzy sets and fuzzy numbers has been quickly applied in technology and informatics, the most practical, engineering.

Expert systems are based almost totally on fuzzy logic. Fuzzy logic is used in real time systems which control technological processes.

Theory of fuzzy logic and fuzzy sets are applied also in economy and social science.

2 Modeling basing on fuzzy logic

To control technological process or device we need mathematical model.

Model design is difficult task. The more complex process or device the more difficult is to design the model. The precise description of complexity is impossible and simplifying is required.

The use of the theory of fuzzy sets makes able creation of fuzzy model of the object or system which represents essential features. The most important feature is coding of information on the base of fuzzy sets and not by sharp value of classic math.

We can obtain greater accuracy of the model and its structure simplifying by modeling which bases on fuzzy logic using certain set of linguistic variables and rules "IF…. THEN…." which are qualitative description of the system very close to the natural language.

Models based on fuzzy logic are characterized by simplicity and flexibility of their structure and great effectiveness. Creation of the rule base is simple and natural.

The work of the decisive system basing on the fuzzy logic depends on the definition of fuzzy rules which were introduced into the rules base.

There are following types of rules:

IF a is A1 AND b is B1 THEN c is C1

IF a is A1 AND b is not B1 THEN c is C2

where:
a, b, c are linguistic variable
A1, …, C2 are fuzzy sets reflecting linguistic variables

The most important feature which differentiates fuzzy sets from classic rules type IF – THEN is application linguistic variables described by fuzzy sets using degree of membership.

It is easy to write using fuzzy logic methods by conditional expressions with short-circuit logical forms.

Linguistic "is" denoted membership to certain fuzzy sets. Logical connective "and" can be replaced by operator T-norm (e.g. minimum operator, etc.). Rules can describe models with many inputs and one output as well as many outputs.

Filling the rules "IF …, THEN" can be obtained from different sources: expert knowledge, quality modeling or information gathering algorithm.

3 Fuzzy inference concept

Pattern of fuzzy inference based on data processing of quantitative variables into linguistic variables then system modeling on the base of the rule's base which denoted design engineer knowledge about the system and on the end, processing back to quantitative variable.

This model is typical fuzzy logic controller which received crisp values describing device status or technical system and process them into sharp value which steers such system.

It consist of four parts:

1. fuzzy block
2. rules base
3. inference block
4. defuzzification block

3.1 Fuzzification

Input data meet fuzzyfication block in which they are fuzzified. Data fuzzificaton consist in assigning of membership degree to certain fuzzy sets. Each of these sets is definite by linguistic variable.

In order to obtain fuzzy variables it is important that membership functions of each fuzzy sets were described by mathematical formula.

We obtain output variables of fuzzification block by substitution of input values to membership function.

3.2 Rules

Rules base it is a linguistic model which is set of fuzzy rules $R^{(k)}$, k=1, …,N denoted:

$R^{(k)}$: **IF** (x_1 is A_1^k **AND** x_2 is A_2^k … **AND** x_n is A_n^k)

THEN (y_1 is B_1^k **AND** $\mathbf{y}_2$ is B_2^k … **AND** y_m is B_m^k)

where:

N – number of fuzzy rules

A_i^k - fuzzy sets, which: supp (A_i^k) $\subseteq$ $\mathbf{X}_i$ $\subset$ $\mathbf{R}$, i=1, …, n_k

B_j^k - fuzzy sets, which: supp (B_j^k) $\subseteq$ $\mathbf{Y}_j$ $\subset$ $\mathbf{R}$, j=1, …, m_k

$x_1, x_2, \ldots, x_n$ - input variables of linguistic model (linguistic variables)

$\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbf{X}_1 \times \mathbf{X}_2 \times ,\ldots, \times \mathbf{X}_n$

$y_1, y_2, \ldots, y_m$ - input variables of linguistic model (linguistic variables)

$\mathbf{y} = (y_1, y_2, \ldots, y_m) \in \mathbf{Y}_1 \times \mathbf{Y}_2 \times, \ldots, \times \mathbf{Y}_m$

$\mathbf{X}_i$, i=1, …,n - universes of input variables

$\mathbf{Y}_j$, j=1, …, m – universes of output variables

R – real number set

Simplyfied notification $A_1^k \subseteq \mathbf{X}_i$ and others similar means, that carrier of fuzzy set A_i^k is subset of the $\mathbf{X}_i$ set.

Model assumptions:

All rules $R^{(k)}$, k=1, …, N are connected each other by logic operator "or".
Output variables $y_1, y_2, \ldots, y_m$ are independent.
Independence of output variables is the reason that in this paper fuzzy rules are consider as a scalar output in following form:

$R^{(k)}$: **IF** (x_1 is A_1^k **AND** x_2 is A_2^k … **AND** x_n is A_n^k)

THEN (y is B^k)

where:

supp(B^k) $\subseteq \mathbf{Y} \subset \mathbf{R}$, k = 1,2, …, N

Fuzzy rules consists of part so called predecessor (IF…) and part THEN(…) so called successor. In predecessor is set of all conditions and in successor is conclusion.

Denoting:

$$\mathbf{X} = \mathbf{X}_1 \times \mathbf{X}_2 \times, \ldots, \times \mathbf{X}_n$$

$$A^k = A_1^k \times A_2^k \times, \ldots, \times A_n^k$$

It is possible to show fuzzy rule $R^{(k)}$ as a implication in the form:

$$R^{(k)} : A^k \Rightarrow B^k, \quad k = 1, \ldots, N$$

Rule $R^{(k)}$ is fuzzy implication specified on the set $\mathbf{X} \times \mathbf{Y}$. It means that $R^{(k)} \subseteq \mathbf{X} \times \mathbf{Y}$ is a fuzzy set in the following membership function:

$$\mu_{R(k)} (\mathbf{x}, y) = \mu_{Ak \Rightarrow Bk} (\mathbf{x},y)$$

During designing of the fuzzy inference model one must proposed certain adequate number of consistent inference rules, sufficient from the modeling problem point of view.

3.3 Inference block

Inference block has in input fuzzy set:

$$A' \subseteq \mathbf{X} = \mathbf{X}_1 \times \mathbf{X}_2 \times ,\ldots, \times \mathbf{X}_n$$

Which we obtain as a result of fuzzification input data:

$$\mathbf{x}^* = (x^*_{1,} x^*_2 ,\ldots, x^*_n) \in \mathbf{X}$$

As an output of fuzzification blok we obtain fuzzy sets according to fuzzy inference rule *modus ponens* :

Premise

$$\mathbf{X} = (x_{1,} x_{2,\ldots,} x_n) \text{ is } A'$$

$$A' = A_1' \times A_2' \times ,\ldots, \times A_n'$$

Implication

$$R^{(k)}: A^k \Rightarrow B^k , k=1, \ldots, N$$

$$A^k = A_1^k \times A_2^k \times ,\ldots, \times A_n^k$$

Conclusion

$$y \text{ is } B^{*k}$$

As an output of inference block we obtain N fuzzy sets B^{*k}.
Fuzzy set B^{*k} is a composition of fuzzy set A' and relation of fuzzy implication $R^{(k)}$:

$$B^{*k} = A' \circ (A^k \Rightarrow B^k), \quad k = 1, \ldots, N$$

Membership function of fuzzy set B^{*k} is as follows:

$$\mu_{B^*k}(y) = \sup_{x \in X} [\mu_{A'}(\mathbf{x}) \overset{T}{*} \mu_{A^k \Rightarrow B^k}(\mathbf{x}, y)]$$

Membership function form $\mu_{B^*}{}^{k}$ (y) depends on method of calculation of Cartesian product of fuzzy sets A', T-norm and method of calculation of fuzzy implication $R^{(k)}$.

The conclusion of this scheme is the result of local reposition of the premise A' and the rule $R^{(k)}$, k = 1, …, N and then aggregation of resulting fuzzy sets.

As a result of aggregation of rules $R^{(k)}$, k = 1, …, N we obtain fuzzy set B' with membership function:

$$\mu_{B'}(y) = \max_{1 \le k \le N} \mu_{B^*}{}^{k}(y)$$

Such approach is a local approach to a problem to synthesis of inference block.

3.4 Defuzzification block

Output value of the inference block is N fuzzy sets B^{*k} together with membership functions $\mu_{B^{*k}}$ (y) , k = 1, …, N, which aggregate in one fuzzy set B' with membership function $\mu_{B'}$ (y).
The task of defuzzification block is fuzzy sets B^{*k} (or fuzzy set B') transformation into one crisp value $y^* \in \mathbf{Y}$.

We can calculate fuzzification block output using different methods, The most using methods are:

1. Center average defuzzification method
2. Center of gravity method
3. Mean of maximum method
4. Height defuzzification method

4 Fuzzy logic application for assessment of company activity

Evaluation of the activity of the company is based on many different criteria in conditions of not certain information expressed by natural language.

Economy science uses many non sharp names which classic mathematics is not able to process.

Empirical researches confirm the fact that auditors have evaluated companies in natural language which operate qualitative values mostly.

Qualitative description gives fuzzification of objects membership to sets.

One of the solution of above mentioned (quality) problems is use of fuzzy sets theory and fuzzy logic for assessment of economic activity of the company. Fuzzy set limits are not sharp what makes possible modeling notions which are not univocal (unique) and not crisp.

4.1 Fuzzy model of assessment of the company activity

Model shows application of fuzzy sets theory and fuzzy logic for evaluation of company activity. For simplification there are chosen only three indexes as a base for assessment of company activity.

Input variables:

- the fluency indicator
- the profitability indicator
- the debts indicator

Assessment of company activity is presented by finance condition indicator.

Output variable:

- the finance condition indicator

Indicators of evaluation of company activity are described by linguistic variables. Membership functions for each / respective linguistic variable terms are described by trapezoidal fuzzy numbers.

WP – fluency indicator

Linguistic variable: *Fluency* = { V*ery low, Low, Average, High}*

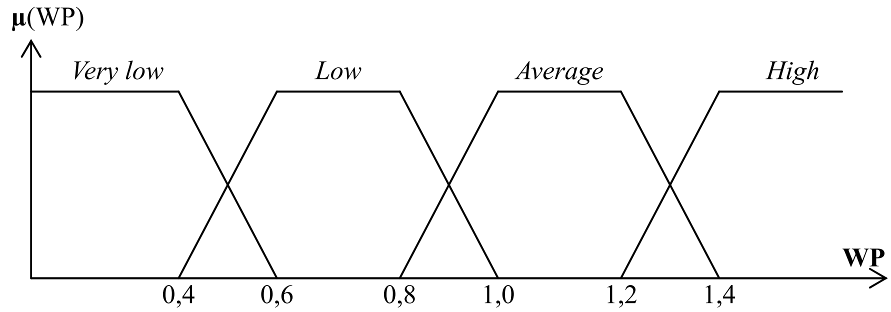

Figure 4.1 Exemplary membership functions of linguistic variable *Fluency*

WR - profitability indicator

Linguistic variable: *Profitability* = { *Small, Average, Big, Very big* }

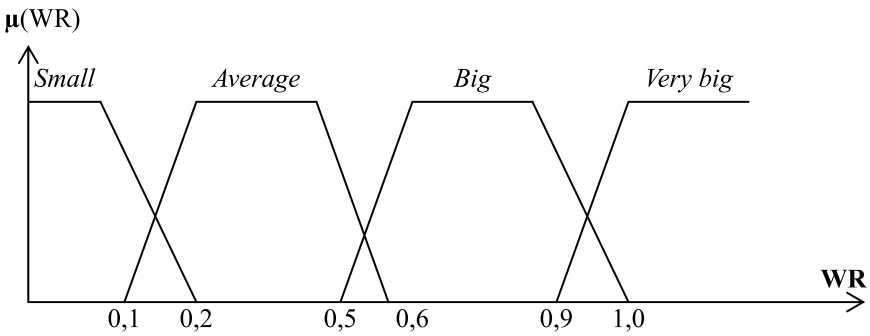

Figure 4.2 Exemplary membership functions of linguistic variable *Profitability*

WZ – debts indicator
Linguistic variable: *Debts* = { *Small, Average, Big* }

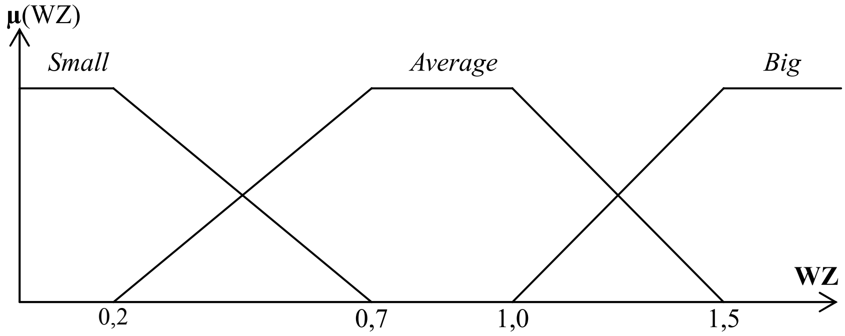

Figure 4.3 Exemplary membership functions of linguistic variable *Debts*

WK – company finance condition indicator
Linguistic variable : *Finance Condition* = { *Low, Average, High* }

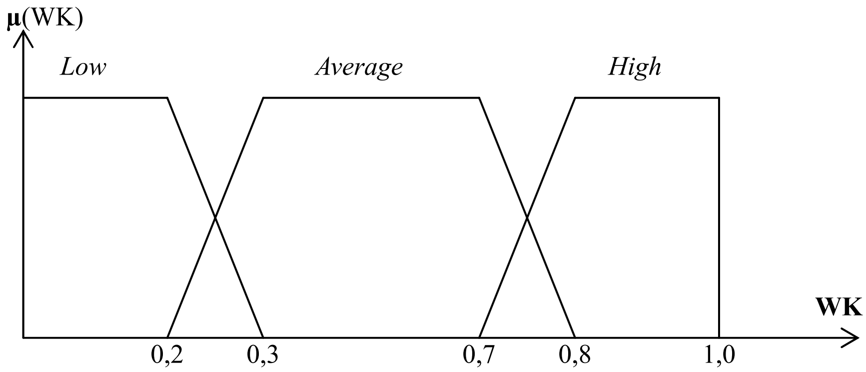

Figure 4.4 Exemplary membership functions of linguistic variable *Finance Condition*

Inference rules:

R^1: **IF** *Fluency* = *Average* **and** *Profitability* = *Average* **and** *Debts* = *Average* **THEN** *Finance Condition* = *Average*

R^2: **IF** *Fluency* = *Very low* **or** *Low* **and** *Profitability* = *Small* **or** *Average* **and** *Debts* = *Average* **or** *Big* **THEN** *Finance Condition* = *Low*

R^3: **IF** *Fluency* = *High* **and** *Profitability* = *Big* **or** *Very Big* **and** *Debts* = *Small* **THEN** *Finance Condition* = *High*

All possible input and output states are predicted in this rules of inference. Different input state implicates different output. There are only 3 rules of inference constructed. On the other sets of input values we infer nothing about output. For example, if Fluency=High, Profitability=Average and Debts=High we can infer nothing about Condition.

4.2 Fuzzification

Assuming that input variable in this model have following values:
WP= 0,85
WR= 0,51
WZ= 1,3

As a result of fuzzification process we obtain values of membership function of linguistic variables as input data:

1. Input variable: fluency indicator WP = 0,85

$\mu_{Fluency=Low}$ (0,85) = **0,75**
$\mu_{Fluency=Average}$ (0,85) = **0,25**

$\mu_{Fluency=Low\ or\ Very\ low}$(0,85) =
max{ $\mu_{Fluency=Low}$ (0,85); $\mu_{Płynność=\_Very\ low}$ (0,85)} = max { 0,75; 0} = **0,75**

$\mu_{Fluency=High}$ (0,85) = **0**

2. Input variable: profitability indicator WR= 0,51

$\mu_{Profitability=Average}$ (0,51) = **0,9**
$\mu_{Profitability=Big}$ (0,51) = **0,1**
$\mu_{Profitability=Small\ or\ Average}$ (0,51) =
= max{ $\mu_{Profitability=Small}$ (0,51); $\mu_{Profitability=Average}$ (0,51) } = max {0; 0,9} = **0,9**

$\mu_{Profitability=Big\ or\ Very\ big}$(0,51)=
= max{$\mu_{Profitability=Big}$ 0,51); $\mu_{Profitability=Very\ big}$ (0,51)} = max { 0,1; 0} = **0,1**

3. Input variable: debts indicator WZ=1,3

$\mu_{Debts=Average}$ (1,3) = **0,4**
$\mu_{Debts=Big}$ (1,3) = **0,6**
$\mu_{Debts=Average\ or\ Big}$ (1,3) =
= max { $\mu_{Debts=Average}$ (1,3); $\mu_{Debts=Big}$ (1,3) } = max { 0,4 ; 0,6} = **0,6**
$\mu_{Debts=Small}$ (1,3) = **0**

4.3 Inference Block

It is assumed that T-norm, Cartesian product and fuzzy implication are defined by minimum operation. Operator max – min was used during inference process according to above mentioned rules.

The graphical interpretation of fuzzy inference is shown in Fig. 4.5.

We determine minimum of steps of membership respective premise for each rule. The result is truth level of satisfy a premise. The application of implication method consist in change of shape of membership function of fuzzy set according to level of satisfy a premise by cutting of membership function. The result of this operation is fuzzy sets which correspond with each output value in conclusion.

We obtain:

Rule 1:

$\mu_{B*1}(WK)= \min\{\min\{\mu_{Average}(WP=0,85);\mu_{Average}(WR=0,51);\mu_{Average}(WZ=1,3)\}; \mu_{Average}(WK)\} =$

$= \min \{ \min (0,25; 0,9; 0,4), \mu_{Average}(WK)\}= \mathbf{\min\{0,25; \mu_{Average} (WK)\}}$

Rule 1 conclusion is illustrated by set $\mathbf{B}^{*1}$

Rule 2:

$\mu_{B*2}(WK)=\min\{\min\{\mu_{Very\_low\_or\_low}(WP=0,85); \mu_{Small\_or\_Average}(WR=0,51); \mu_{Average\_or\_Big}(WZ=1,3)\}; \mu_{Low}(WK)\} =$

$= \min \{\min \{0,75;0,9;0,6\}; \mu_{Low} (WK)\} = \mathbf{\min \{0,6; \mu_{Low} (WK)\}}$

Rule 2 conclusion is illustrated by set $\mathbf{B}^{*2}$

Rule 3:

$\mu_{B*3}(WK)=\min\{ \min \{ \mu_{High} (WP=0,85); \mu_{Big\_or\_Very Big} (WR=0,51); \mu_{Small} (WZ=1,3)\}; \mu_{High} (WK)\} =$

$=\min\{\min\{0; 0,1; 0\}; \mu_{High} (WK)\} = \mathbf{\min \{0, \mu_{High} (WK)\} = 0}$

Rule 3 conclusion is illustrated by set $\mathbf{B}^{*3}$ (empty set).

The next step is aggregation of all output, it means gathering output sets using sum operator.

It is set B' which is the sum of output sets: B^{*1}, B^{*2}, B^{*3}:

$$B' = B^{*1} \cup B^{*2} \cup B^{*3}$$

$$\mu_{B'}(WK) = \max\{ \mu_{B^{*1}}(WK), \mu_{B^{*2}}(WK), \mu_{B^{*3}}(WK)\}$$

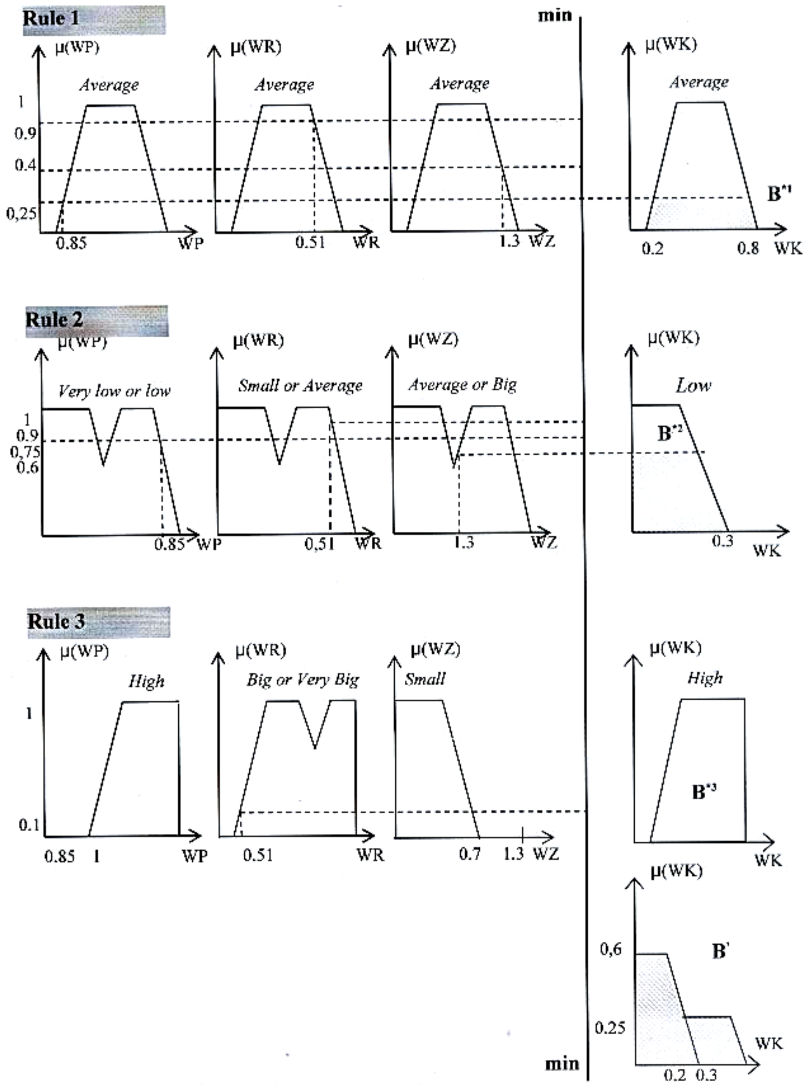

Figure 4.5 Graphic interpretation of fuzzy inferential rules

4.4 Defuzzification

The result of inference has fuzzy form. In order to obtain crisp value we use method of result defuzzification which based on maxima average of output sets.

The choice of the method depends on each particular problem which is considered.

Center average defuzzification method is quick to implement it easy in algorithms.

$$y^* = \frac{0.1 * 0.6 + 0.5*0.25}{0.6+0.25} = \frac{0.185}{0.85} = 0.22$$

The result says that Finance Condition indicator WK equals 0.22

It means that company finance condition is:

$$\mu_{Condition = Low}\ (0.22) = \mathbf{0.8}$$

Company finance condition is low in grade 0.8

$$\mu_{Condition=Average}\ (0.22) = \mathbf{0.2}$$

Company finance condition is average in grade 0.2

$$\mu_{Condition = High}\ (0.22) = \mathbf{0}$$

Company finance condition is high in grade 0.

The interpretation of defuzzification by *Center average defuzzification* method is shown in Fig. 4.6.

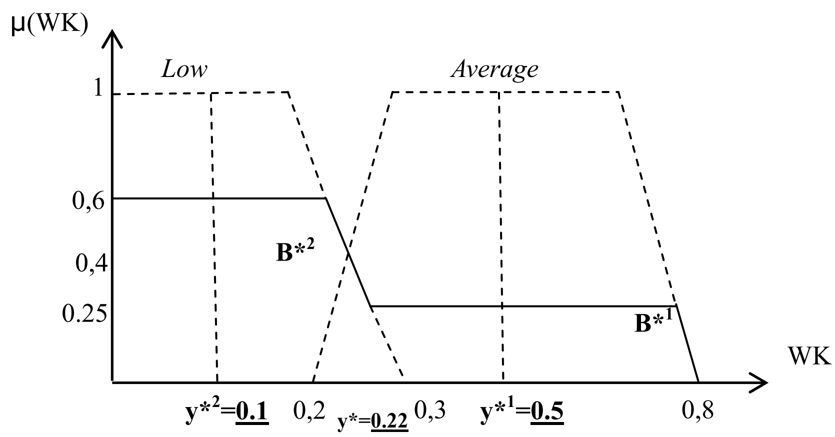

Figure 4.6 Aggregated output *Finance Condition.* Defuzzification by Center Average metod

5 Summary

The model of the assessment of companies efficiency is presented in this paper. This assessment is expressed by company condition indicator.

Presented model has only 3 variables. Full model should take into consideration all important factors including qualitative. Fuzzy sets theory enables it.

Presented model makes possible representation of different indexes of company evaluation and aggregation of evaluations connected with individual indexes at the same time.

Representation of relations between indexes using inference fuzzy rules enables expressing of complex dependence between evaluation indexes and resulting index of company finance condition natural and easy to understanding way.

The advantage of this model is its versatility. It can be used for evaluation of different companies with different profiles of production thanks to standardization of space of considerations by indexes and possibility of aggregation of qualitative different aspects of evaluation of each company activities.

Information obtained on the base of fuzzy inference rules can be helpful during process of taking decisions.

Presented model can be used for creation of more complex models, based on fuzzy logic, artificial neural networks and genetic algorithms in order to improve description of the real.

It is advisable to apply this model in practice through undertaking a study in order to define forms of membership functions for linguistic variables, to define optimal method of defuzzification and to create of rules base for all essential factors of evaluation of company activities.

References

[1] Bojadziev G., Bojadziev M.: *Fuzzy sets, fuzzy logic, applications,* Singapore: World Scientific 1998.
[2] Czogała E., Pedrycz W.: *Elementy i metody teorii zbiorów rozmytych,* PWN 1985.
[3] Driankow D., Hellendoorn H., Reinfrank M.: *Wprowadzenie do sterowania rozmytego,* WNT 1996.
[4] Łachwa A.: *Rozmyty świat zbiorów, liczb, relacji, faktów, reguł i decyzji,* Warszawa: Exit 2001.
[5] Kacprzyk J.: *Zbiory rozmyte w analizie systemowej,* PWN 1986.
[6] Piegat A.: *Modelowanie i sterowanie rozmyte,* Warszawa: Exit 2003.
[7] Rutkowska D., Piliński M., Rutkowski L.: *Sieci neuronowe, algorytmy genetyczne i systemy rozmyte,* Warszawa: PWN 1997.
[8] Yager R.R., Filev D.P.: *Podstawy modelowania i sterowania rozmytego,* Warszawa: WNT, 1995.
[9] Zadeh L. A.: *Fuzzy Sets,* Information and Control, 1965.