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Cluster analysis on the example of work data of the National Power System. Part 2. Research and selected results

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Abstract. The work is a continuation of the article under the same main title and subtitle Part 1. Comparative study of methods and conditions. This article concerns the cluster analysis, which was carried out on the example of data concerning the operation of the National Power System, namely the total generations of nCDGUs and CDGUs listed by PSE Operator. Two algorithms were used to obtain the results of the cluster analysis, i.e. the Ward algorithm and the algorithm of self-organizing two-dimensional topographic maps. The obtained results were interpreted and their discussion and interpretation were conducted.

Keywords. Cluster analysis, National Power System, MATLAB and Simulink, Ward's algorithm, self-organizing two-dimensional maps.

1. Introduction

The cluster analysis based on the National Power System (NPS) database was carried out using two methods, i.e. the non-hierarchical cluster analysis method called the Self-organizing Artificial Neural Networks (SOM) method and the hierarchical method called the Ward's method, belonging to the group of agglomeration methods. The specific properties of Ward's method relate to the pursuit of similar data to concentrate around the mean, and the specific properties of ANN SOM are related to the projection of knowledge on multidimensional topographic maps [4-7, 12-14, 16, 18, 28].

1.1. An example of an analysis using the Self-Organizing Neural Networks method

During cluster analysis using the method of Self-organizing Artificial Neural Networks (SOM), the regularities in the set of input values are projected in the form of clusters onto a topological map, e.g. two-dimensional. The result of learning the SOM network can also be other dimensions, including: a line or every figure in a plane, in space, or in the nth dimension [1, 7-10, 17, 19-24, 28]. In order to demonstrate the functioning of the SOM network method, the data shown in Fig. 1 were used, that is:

1 000 random values from the <-1 ÷ 1> range for the X and Y coordinates.

These data were generated in the MATLAB and Simulink environment using the P = rands (2, 1 000) function, and the results using the function:

plot(P(1,:), P(2,:), '+b') [6, 18, 31-32].

For the projection of the cluster analysis results, a layer consisting of 30 neurons distributed on a grid placed on a plane with dimensions of 5 x 6 neurons was adopted. The expected behavior was to generate a separate response of each neuron to individual areas of the formed neuron network, with individual clusters of neurons cooperating with adjacent clusters.

The following function was used to create a neuron network:

net = newsom([0 1; 0 1],[5 6]),

the structure of the function is the data area and the size of the neuron grid [6, 18, 31] (Figure 1).

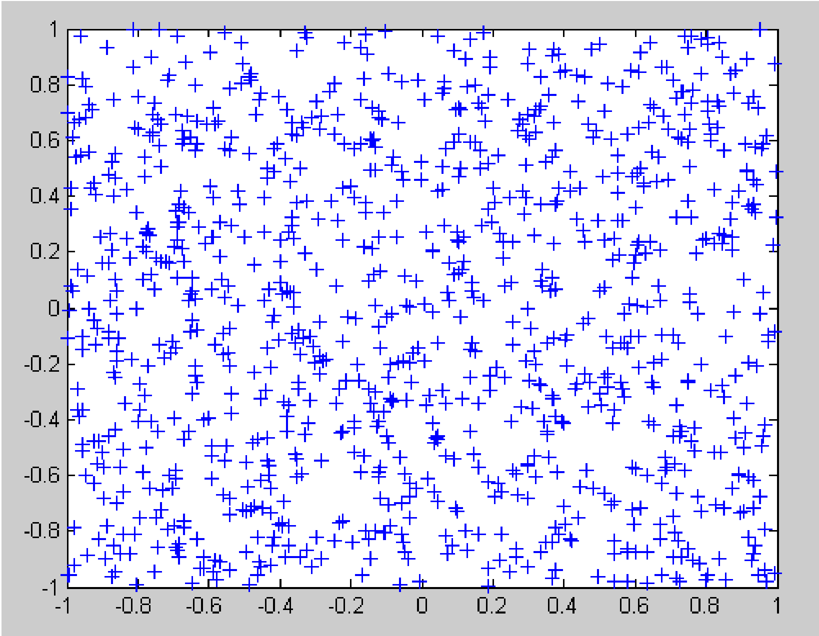

Figure 1. Generated data on the operation of the PPS in 2018. Source: Own study in the environment of MATLAB and Simulinek [6, 18, 31].

The visualization of the SOM network in the initial phase was generated using the function:

plotsom(net.iw{1,1}, net.layers{1}.distances),

the function structure is: weight matrix size, distance matrix size and neighborhood radius. Each neuron is represented by a red dot at the coordinates of its two weights. Initially, all neurons are of equal weight and are concentrated in the center of the area.

The next step is to train the SOM network based on a thousand input vectors over a period of one epoch and re-visualize the weights of the SOM network in a new graph. After the learning process, it can be observed that the layer of neurons began to self-organize, so that each of the neurons classifies a different cluster of the input value space, and the clustered neurons respond to adjacent clusters of neurons appearing on the topological grid (Figure 2).

In order to obtain the projection of neurons onto a two-dimensional map, the following functions available in the MATLAB environment were used [31]:

net.trainParam.epochs=1;

net=train(net,P);

plotsom(net.iw{1,1},

net.layers{1}. distances.

The last step is to identify the most appropriate neuron in each cluster of neurons. To find such a neuron you can use the function:

p = [0.5; 0.3];

a = sim(net, p),

where:

- p - defined class,
- a - a variable that takes the result of the sim function [31].

1.2. An example of cluster analysis using the Competitive Learning method

Artificial Neural Networks are generally divided into Multilayer Unidirectional, Recursive, and Cellular. First, the ANN architecture is designed, its parameters are selected, then it is learned and tested, and finally it is verified [1-3, 15, 17-18] normalization [20-25]. Neurons in the competitive layer learn to represent different clusters of the input value space. The letter P denotes the test set consisting of randomly generated but clustered data. The points are plotted in Figure 3, and a competitive Artificial Neural Network was used to classify these points. In order to perform the first step, the functions available in the MATLAB environment (Figure 4) [6, 18-23, 28, 31-32] were used.

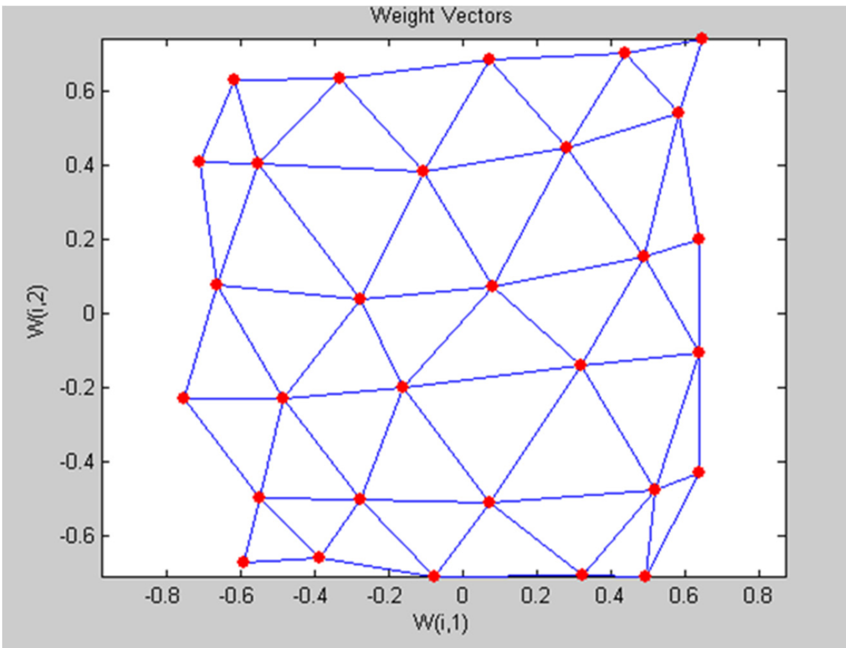

Figure 2. Graph visualizing the last step in creating a neuron mesh Markings: W(i, 1) - weight of the X axis, W(i, 2) - weight of the Y axis, Source: Own elaboration in MATLAB and Simulink environment [6, 18, 31].

Creation of set P:

```
X = [0 1; 0 1];
clusters = 8;
points = 10;
std_dev = 0.05;
P = nngenc(X, clusters, points, std_dev);
```

where:

- X - the scope intended to designate cluster centers.
- clusters - a variable that stores information about the number of clusters.
- points - a variable that indicates the number of points in each cluster.
- std_dev - the standard deviation value for each cluster.
- P - test set.

Generating a plot for set P:

```
plot( P(1, :), P(2, :), '+b');
```

The next step is to use the *newc()* function, which takes three input arguments: an Rx2-sized matrix that stores the maximum and minimum values for the input elements R, the number of neurons, and the learning rate. The initial learning phase of the initiated sample is shown by the graph in Figure 4. The weight vectors were trained in such a way that they appeared in the middle of the input vector clusters. The following functions were used to complete this step:

```
net = newc([0 1;0 1], 8, .1);
w = net.IW{1};
plot(P(1, :), P(2, :), '+b');
```

hold on;

circles = plot(w(:, 1) ,w(:, 2), 'or');

where:

- net - trained network,
- w - fixed weights,
- circles - a variable with the parameters of the drawn plot.

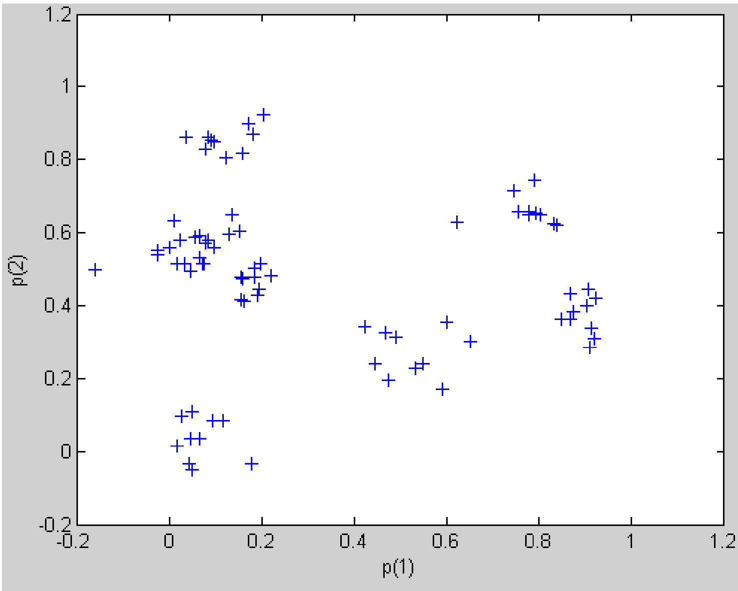

Figure 3. Graph showing the generated data. Symbols: p (1) - value for points on the X axis, p (2) - value for points on the Y axis. Source: Own elaboration in MATLAB and Simulink environment [6, 18, 31].

The next step is to set the number of epochs to train the competing layer. After the following functions were performed, neurons marked with circles moved to the centers of the input vectors located in separate classes (Figure 5).

net.trainParam.epochs = 7;

net = train(net,P);

w = net.IW{1};

delete(circles);

plot(w(:, 1),w(:, 2), 'or');

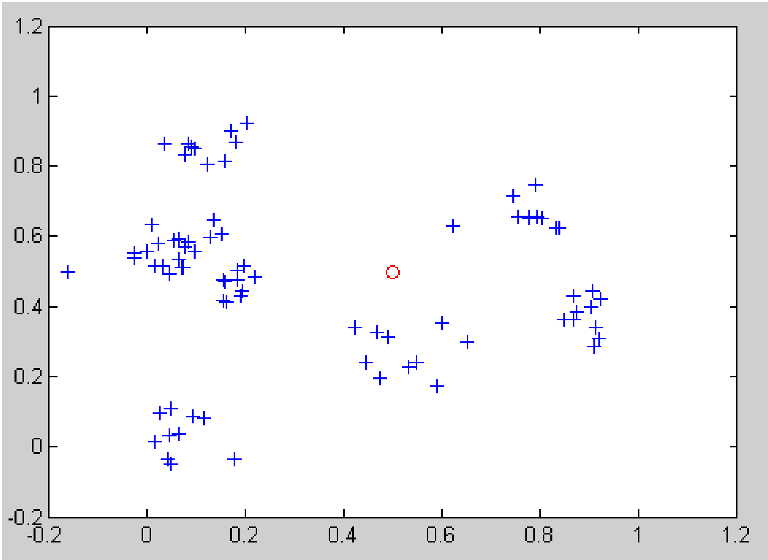

Figure 4. Graph of the training attempt initiated. Source: Own study in MATLAB and Simulink environment [6, 18, 20, 31].

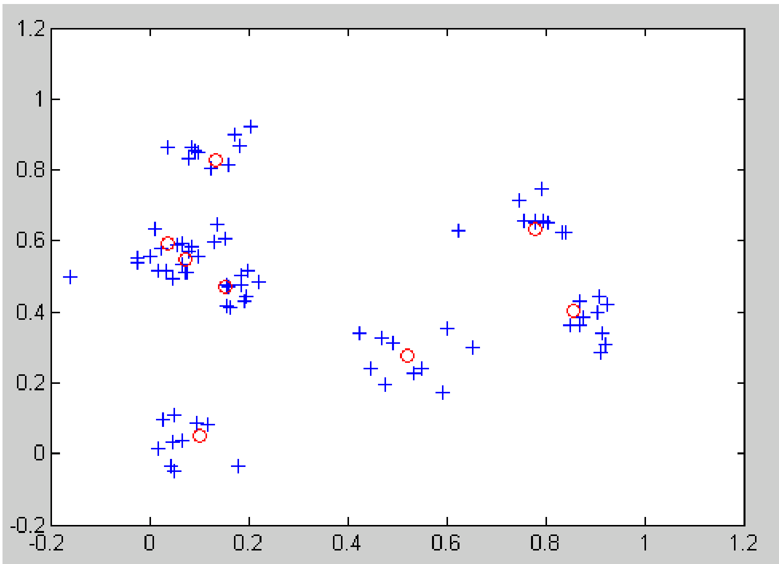

Figure 5. Chart showing classified data. Source: Own study in MATLAB and Simulink environment [6, 18, 20, 31].

The final activity is to use the competitive layer as a classifier where each neuron inspires a different category. In this case, the input vector with values in the range [0; 0.2].

The result marked with the letter a indicates a neuron that is responsible for the class to which specific input data belong, but it should be taken into account that the SIM function returns the result in a sparsely distributed matrix for competing layers:

p = [0; 0.2];

a = sim (net, p),

where:

p - considered input parameter.

a - a result indicating the nearest neuron and input class.

Ultimately, the result was a = (1, 1).

2. Analysis of the work data of the National Power System

2.1. Two-dimensional self-organizing topological maps

In order to conduct cluster analysis using the SOM Artificial Neural Network, numerical data on the operation of the National Power System in the Polish Power Grid subsystem [30] were used, including cluster analysis for the data on the total generation of nJWCD and the total generation of JWDC (Figure 6).

The total generations are summarized into points representing the ratio of the total nCDG generation to the total JWCD generation as the hourly reading in 2014, with 8,760 points in the chart [6, 18, 20, 31].

In order to select the optimal SOM Artificial Neural Network learning parameters, the following parameters were set:

netp1 = newsom ([0 20000; 0 10000], [5 7]) - a network of 35 neurons in a 5x7 mesh for 1 sample,

netp2 = newsom ([0 20000; 0 10000], [3 5]) - a network of 15 neurons in a 3x5 mesh for 2 samples,

net.trainParam.epochs = 7; both trials took place in 7 epochs.

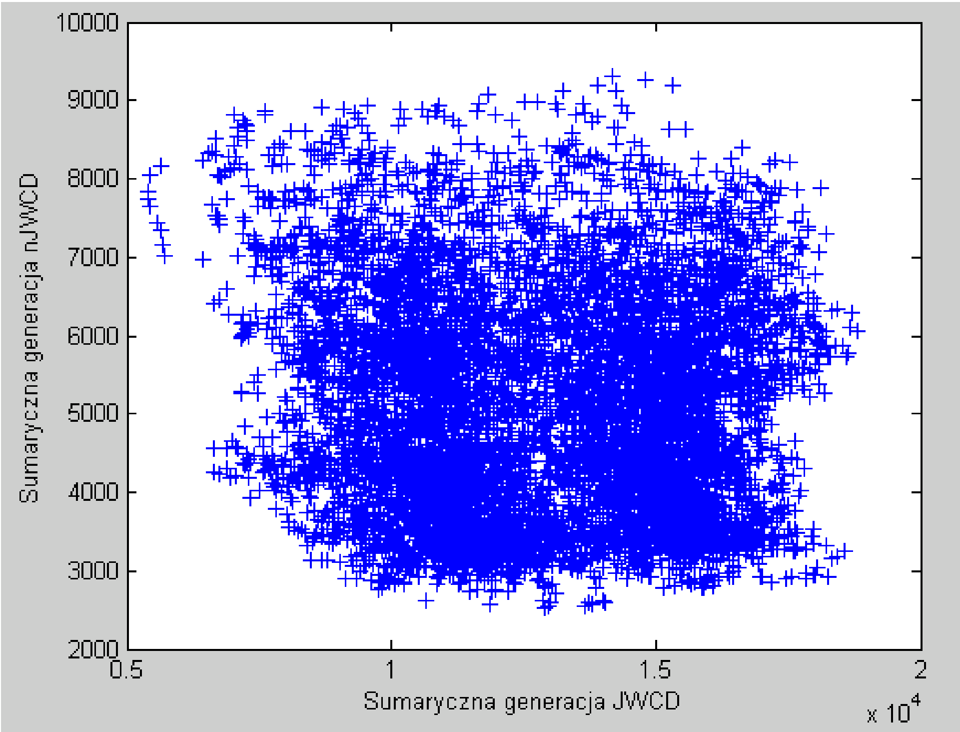

Figure 6. Map of total JWCD and NCDGU generations for 2014 data on the operation of the PPS with regard to the PSE subsystem. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in the MATLAB and Simulink environment [6, 18, 20, 31].

Moreover, two attempts to create neural meshes as in Fig. 7 were shown, which were generated in the hexagonal topology of the neighborhood map.

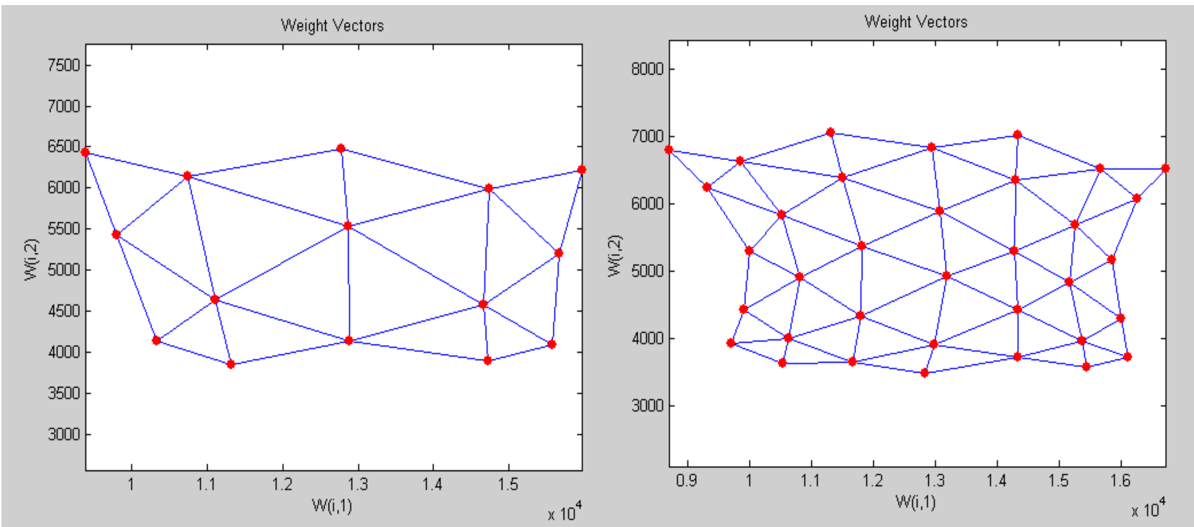

Figure 7. Grids of neurons for summary generations of nJWCD and JWCD. Source: Own study in the environment of MATLAB and Simulink [6, 18, 20, 31].

It can be noticed, among others, that the grids are arranged in a similar shape and show very similar clusters of neurons, therefore, for the analysis of clusters from the range of several years, the parameters used for the grid visible on the left side of the graph in Fig. 8 were used due to the much shorter time calculations and a grid of 15 neurons.

After generating appropriate topological maps, it can be concluded that the main disadvantage of this solution is the computational complexity, which grows very quickly with the increase of the mesh size (i.e. with the number of neurons of the designed experiment) and, consequently, with the increase in the number of epochs and the size of the data set .

This is because the main cost of the algorithm is comparing vectors. In turn, the advantage of such an analysis is a relatively simple visualization and, in interpretation, a very clear visualization of the results.

The next step is to generate the SOM graph and network for the data for each year from 2014-2018 (Figure 8-Figure 12) in separate experiments using the same method, which involved, inter alia, with the separation and standardization of data on the total generations of nJWCD and JWCD.

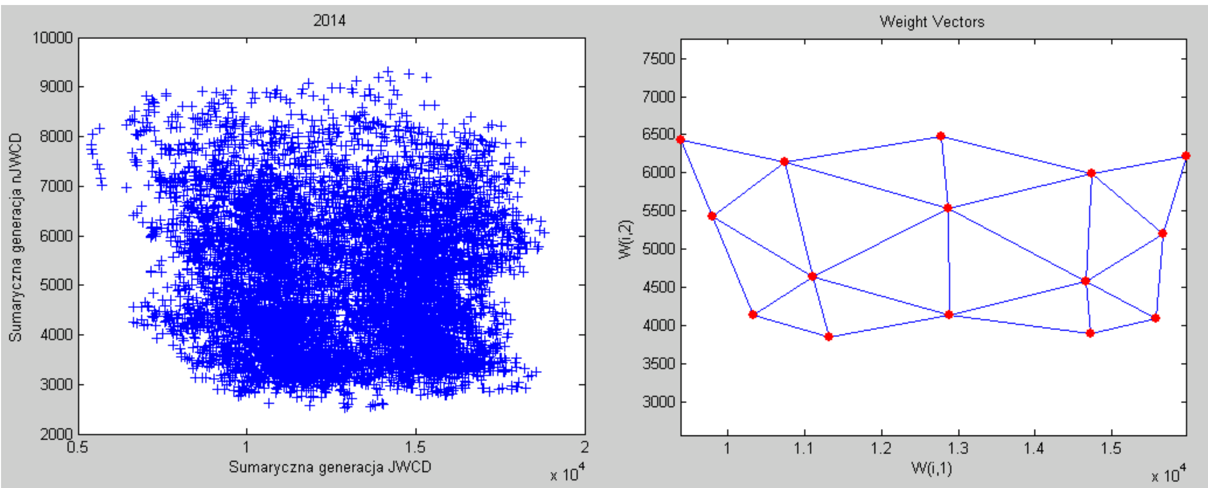

Figure 8. SOM neuron grids generated for the total generations of nJWCD and JWCD for 2014 data. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in MATLAB and Simulink environment [6, 18, 20, 31].

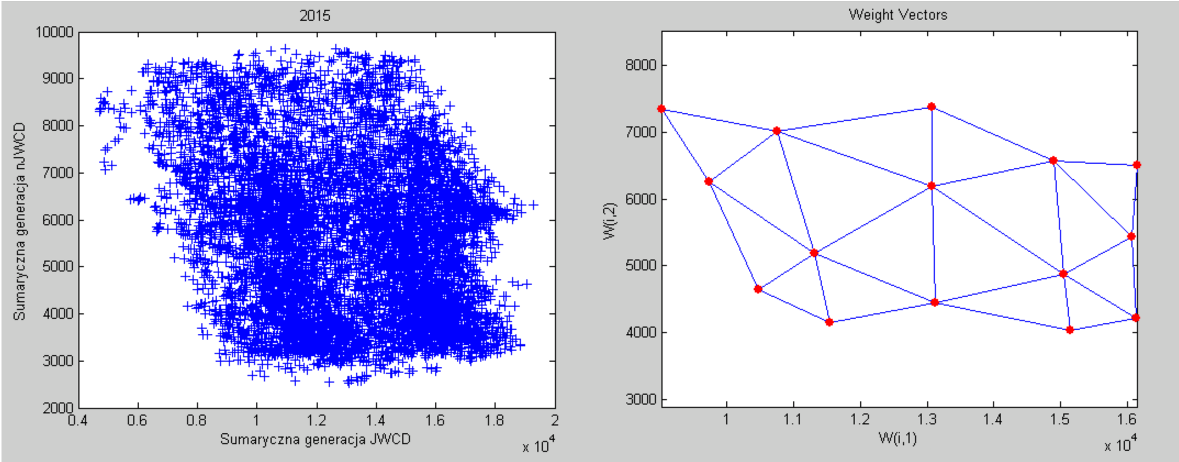

Figure 9. SOM neuron grids generated for the total generations of nJWCD and JWCD for 2015 data. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in MATLAB and Simulink environment [6, 18, 20, 31].

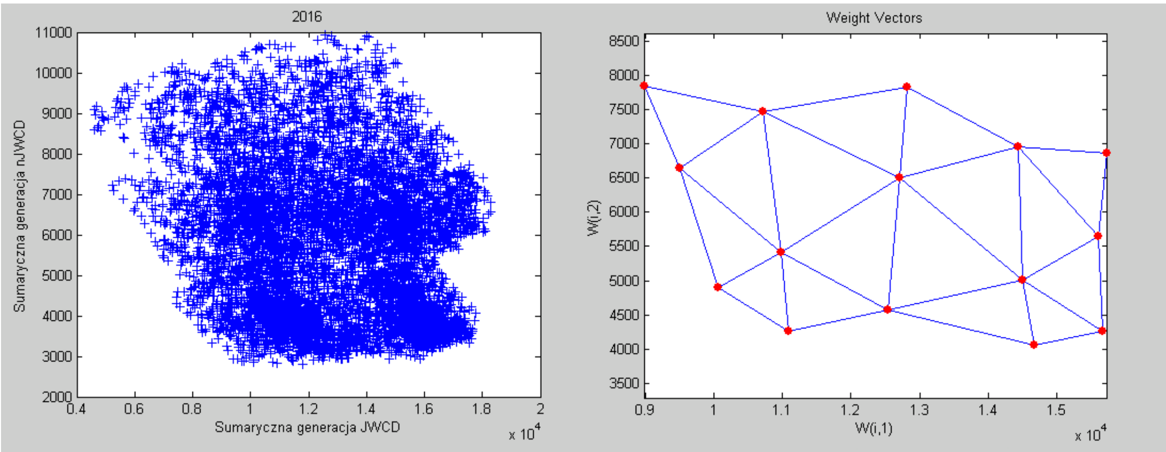

Figure 10. SOM neuron grids generated for the total generations of nJWCD and JWCD for 2016 data. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in MATLAB and Simulink environment [6, 18, 20, 31].

After dumping the input data, an attempt was made to interpret the obtained test results, because the values appearing at the outputs of neurons belonging to the output layer are not always sufficiently understandable, even on the topological map.

From the analysis of the maps and grids shown in Fig. 8-12, it can be seen, inter alia, that with the passage of time the weight w(i, 2) representing the total generation of nCDGU has a wider range of assumed values than the weight w(i, 1), and the grid for each subsequent year is getting closer and closer to the diagonal of the graph running from the upper left corner to the lower right corner, which indicates a regularity confirming the principle that with a decrease in the total value of nCDGU generation, the value of the total generation of CDGU increases. An important observation is also the smaller and smaller distances between neurons located in the lower corners of the mesh.

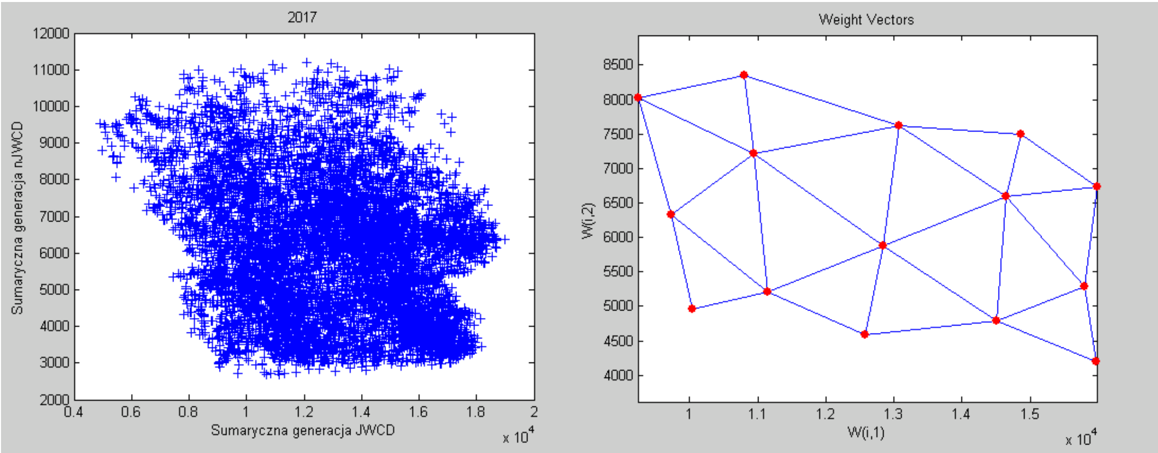

Figure 11. SOM neuron grids generated for the total generations of nJWCD and JWCD for 2017 data. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in MATLAB and Simulink environment [6, 18, 20, 31].

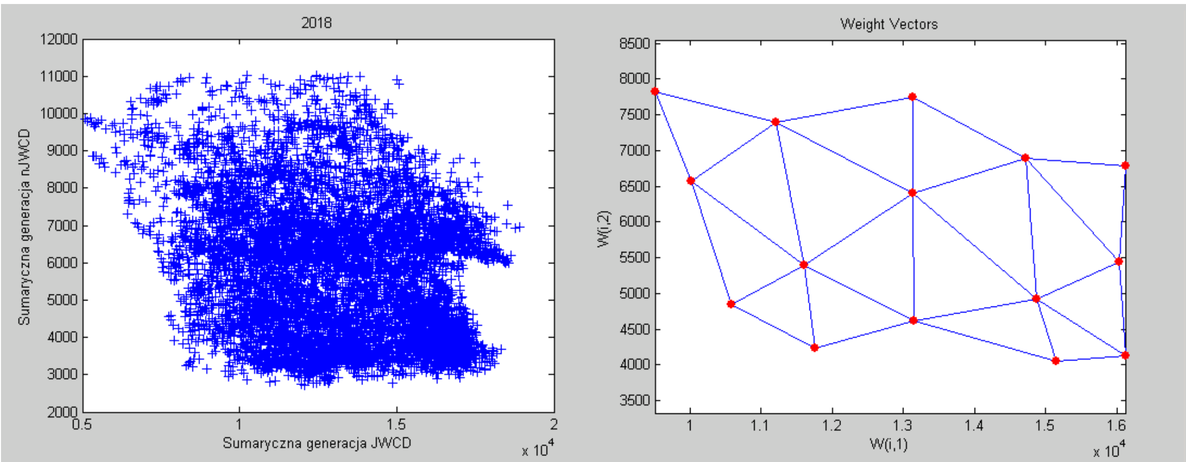

Figure 12. SOM neuron grids generated for the total generations of nJWCD and JWCD for 2018 data. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own elaboration in MATLAB and Simulink environment [6, 18, 20, 31].

3. Hierarchical cluster analysis using Ward's algorithm

Moreover, a hierarchical cluster analysis of the results of the work of the National Power System was carried out with the use of Ward's algorithm. The analysis of variance was used to estimate the distance between the clusters by minimizing the sum of squared deviations in the center of the clusters. The measure of the differentiation of clusters to mean values was the Error Sum of Squares with the measure of the distance form [5-6, 12-14, 26, 28]:

$$EES = \sum_{i=1}^{k} (x_i = \bar{x})^2 \tag{1}$$

where:

x_i - value of the variable representing the segmentation criterion for the i-th object,

k - number of elements in the cluster.

Ward's method is very effective, however, it aims at creating clusters of small size, hence the analysis included data on the PPS operation only for one year, i.e. from 2018. The obtained results are presented in Figure 13, where the graph of points is shown on the left. summary generations of nJWCD and JWCD, and on the right - a dendrogram generated by Ward's method.

In order to create graphs, the data was previously normalized, and then appropriate functions of the MATLAB and Simulink environment, such as e.g. [6, 18, 31]: tree = linkage (X, 'ward'); Creating a tree structure using the Ward's method; D = pdist (X); Function calculating distances between vectors; dendrogram (tree) - create a dendrogram based on the results of the linkage function.

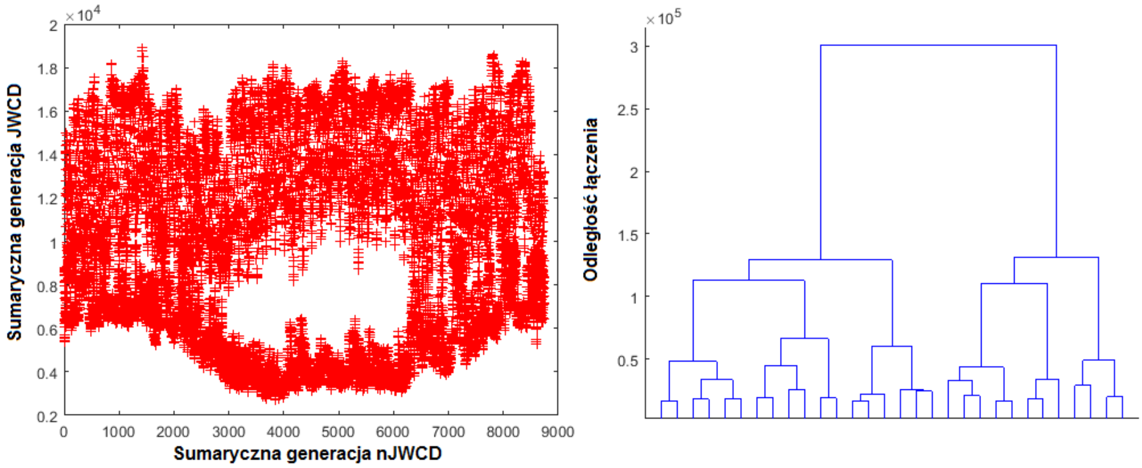

Figure 13. Chart of data on the PPS operation in 2018 and a dendrogram based on them. Descriptions: Sumaryczna generacja JWCD (Pol.) - Total JWCD generation (Eng.), Sumaryczna generacja nJWCD (Pol.)- total nJWCD generation (Eng.). Source: Own study with the use of MATLAB and Simulink environment [6, 18, 31].

As a result of applying the Ward's method, a dendrogram was obtained that shows the hierarchical structure of a set of objects due to the decreasing similarity between them. This method is based on the construction of a graph in the form of a tree. Ward's algorithm belongs to the "bottom-top" family of algorithms, which means that each vector is a separate cluster, and then they are combined into larger and larger clusters until the correct number of clusters, for example specified by the user, or when all vectors qualify as one cluster.

A typical feature of Ward's algorithm is the ability to represent the clustering structure in the form of a dendrogram. In the case of the current analysis, it can be noticed that on the vertical axis of the system into which the dendrogram is inscribed, the values corresponding to the degrees of similarity of the groups measured as distance are presented.

Such a representation of the results makes it possible to evaluate the number of clusters in the case when their number is not known at the beginning, or in the case of outliers [2-3, 5-6, 13-14, 27-29]. Three basic steps were performed to perform the analysis: counting distances, building a tree based on distance, and trimming the tree to narrow down the number of groups.

When analyzing the dendrogram, it can be seen, inter alia, that the data form two large groups divided into three subgroups. Due to the fact that the data comes from daily reports generated by PSE Operator [6, 20, 30], there is a high probability that each cluster represents an appropriate state of operation of the PPS system at certain times, because the data comes from the entire year in an interval of one hours.

Based on the results of the analysis shown in Fig. 14, it can be seen that the distance began to increase rapidly at some point, while for about 70-75% of steps it increased almost linearly, which indicates densely spaced objects, therefore, attention should be paid to smaller clusters formed in the initial phase of the algorithm.

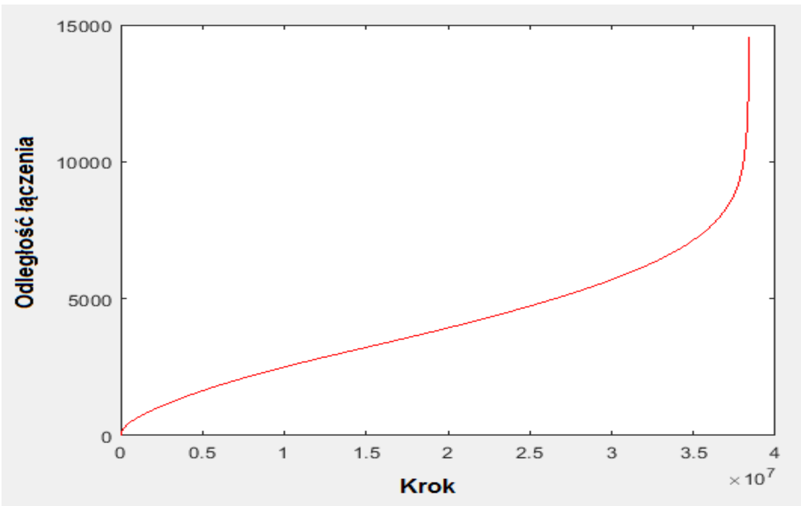

Figure 14. The course of the results of the agglomeration method for Ward's algorithm based on data on the work of JKSE in 2018. Descriptions: Krok (Pol.) – Step (Eng.), Odległość łączenia (Pol.) - Connecting distance. Source: Own elaboration using the Neural Network Toolbox of MATLAB and Simulink [5-6, 18, 20, 28].

The analysis aimed to detect groups in the set of observations within which the elements are similar to each other. Unlike discriminant methods, where the groups were known a priori, Ward's agglomeration method classified the data into previously unknown classes. Efficiency is highly data dependent, and readability decreases as the number of observation steps increases.

4. Discussion of selected research results

The computer-aided data analysis discussed in the work is more and more often used to obtain knowledge from ever larger data sets, or to perform extensive analyzes that consist of many stages, etc. The results of empirical research are much more often used as a set of data that can be used in data mining.

A problem that the analyst encounters most often is large data sets and unclear situations in formulating research hypotheses, when the relationships between the variables and patterns are not known, and the regularities in the values of the input values are not known. Another problem is the numerical complexity of the analyzes carried out, which makes it necessary to use modern computing clusters that provide strong computing power facilities.

However, the cluster analysis from the above-mentioned On the other hand, it turns out to be the most useful method when the structure of data connections is unknown, because its result is the isolation of grouped objects with similar characteristics, which is particularly useful in identifying, describing and interpreting the observed phenomena.

In the conducted research, cluster analysis was used, especially using its two methods: hierarchical with the use of Ward's algorithm in the field of agglomeration methods and non-hierarchical using the method of Self-organizing Artificial Neural Networks with knowledge projection onto two-dimensional topological maps with an overlaid rectangular grid.

The skilful use of complex computational techniques is the main sense of data analysis in research conducted using cluster analysis methods. The methodology and matching appropriate algorithms for specific data, combined with expert knowledge, affects the effectiveness of the developed strategy for data mining and analysis.

5. Final remarks

The work is a continuation of the article under the same main title in part 1 entitled: Comparative study of methods and conditions.

This article shows that both the hierarchical method carried out with the use of Ward's algorithm and the non-hierarchical method carried out with the use of Artificial Neural Networks SOM can be successfully used in wider research on the operation of the National Power System, which was initially tested and shown in this article on data actual data concerning the operation of the Polish Power Transmission Grid, i.e. on the basis of the data on the total generations of nCDGU and JWCD.

The conducted analysis showed that there are many very interesting regularities in the field of data on the operation of the PPS system.

At the same time, the obtained research results and their interpretation were discussed.

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