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The scalarization approach for multi-objective optimization of network resource allocation in distributed systems

Abstract. The paper presents a multi-objective optimization framework to the network resource allocation problem, where the aim is to maximize the bitrates of data generated by all agents executed in a distributed system environment. In the proposed approach, the utility functions of agents may have different forms, which allows a more realistic modeling of phenomena occurring in computer networks. A scalarizing approach has been applied to solve the optimization problem.

Keywords: distributed system, resource allocation, multi-objective optimization

1 Introduction

One of the most important challenges of mathematics and computer science is mathematical modeling of data transmission processes in computer networks. The difficulties in this area result from the fundamental fact that no one understands phenomena occurring in real systems consisting of many interacting elements [6, 18]. Modeling of the data transmission process in computer networks, particularly in the Internet, are experiencing further difficulties due to the fact that this is a highly heterogeneous environment in which many communications protocols are used, and at the lack of centralized governance in either technological implementation or policies for access and usage, the emergence of bottlenecks is imminent [7, 25, 40]. To solve the issue of modeling data flows in computer networks many approaches have been proposed. Some of them use the following fields of science: theory of queues [1, 2, 14], Markov chains [1, 35], various analytical methods [17, 20, 30, 36], deterministic chaos [9, 29, 31, 37], probability calculus [3, 4, 28], Petri nets [11], time series [15], optimization [20], multi-objective optimization [5, 10, 13, 19], and computer simulation [12, 23, 30]. In recent years there has been increasing interest in the use of multi-objective optimization methods to solve the network resources allocation problem. In this respect both exact and approximation methods are used. The exact methods (also called the classical methods) use techniques such as: no-preference methods, a priori methods, a posteriori methods, and physical programming [21]. The approximation methods that use different

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metaheuristics based on evolutionary algorithms [5, 10, 38], and non-standard algorithms using the following techniques: local search, simulated annealing, tabu search, path relinking, scatter search, ant colony optimization, and particle swarm optimization [32] have become very popular. The exact methods are applied when it is possible to assume that data flows in distributed systems are wide sense stationary processes. These methods are effective for small-size problems. When problems become more difficult, usually due to their NP-hard complexity, it is advisable to use approximation methods. These methods are usually used when data flow are subject to rapid and unpredictable changes. In such cases, the approximation methods allow to develop adequate, in the established sense, algorithms used in various aspects of computer networks management [27, 41]. However, they are burdened with a fundamental and irreducible disadvantage. Namely, they make it possible to obtain approximate solutions whose distances from the optimal solutions are unknown and often impossible to determine. This defect does not have the exact methods. However, these methods require making many simplifying assumptions, which may cause that the data flow model may prove to be inadequate.

The subject of the paper is the issue of multi-criteria optimization of network resource allocation in distributed systems. "A distributed system is a software system in which components located on networked computers communicate and coordinate their actions by passing messages. The components interact with each other in order to achieve a common goal. Three significant characteristics of distributed systems are: concurrency of components, lack of a global clock, and independent failure of components" [8]. Distributed systems are usually small scale systems designed to perform established computational tasks. In these systems the same communication protocol is often applied, e.g. TCP [30]. If the data flows in such systems can be regarded as wide sense stationary processes, the problem of optimizing the network resource allocation can be solved by exact techniques.

In the paper for modeling the message passing process the multi-agent approach has been used [27]. It has been assumed that the purpose of each agent is to maximize the transmission rates taking into account the limited bandwidths of network links. To solve the multi-objective optimization problem a scalarizing approach has been applied [21, 22].

The rest of this paper is organized as follows: section 2 describes a mathematical model of a distributed system; section 3 formulates the multi-objective optimization problem and provides some methods to solve it. The final section summarizes the paper conclusions and shows possible directions for future work.

2 Model of a Distributed System

The mathematical model of a distributed system determines the ordered pair,

$$M=(N, D), \tag{1}$$

where: N is a model of the structure, and D is a model of the message passing process.

2.1 Model of Distributed System Structure

The model N , Eq. (1), determines the net,

$$N = (V, E, \mathbf{c}), \quad (2)$$

where:

- $V = H \cup R = \{v_1, \dots, v_{m_V}\}$ is a set of vertices, $H = \{h_1, \dots, h_{m_H}\}$ is a set of vertices representing computers, and $R = \{r_1, \dots, r_{m_R}\}$ is a set of vertices representing routers, $m_V = m_H + m_R$;
- $E = \{e_l = (v_l, w_l) \in V \times V : v_l \neq w_l, l = 1, \dots, m_E\}$ is a set of directed edges representing one-way network connections (or network links) among pairs of different elements of the distributed system structure;
- $\mathbf{c} = (c_1, \dots, c_{m_E})^T \in \mathbb{R}_+^{m_E \times 1}$ is a vector of capacities of all network links.

Let $K = \{1, \dots, m_H\}$ be a set of numbers of computers from set H . Let $L = \{1, \dots, m_E\}$ be a set of numbers of network links from set E . Let $P = \{p_1, \dots, p_{m_P}\}$ be a set of paths connecting all pairs of different computers from set H , where: $p_s = p_{(\alpha_s, \omega_s)} = (\alpha_s, r_s^{(1)}, r_s^{(2)}, \dots, r_s^{(n_s)}, \omega_s)$ is the path linking the pair of computers $(\alpha_s, \omega_s) \in H \times H$, $\alpha_s \neq \omega_s$, $r_s^{(i)} \in R$, for $i = 1, \dots, n_s$, and $s = 1, \dots, m_P$. Let $S = \{1, \dots, m_P\}$ be a set of numbers of all paths from set P .

Rules of the message passing between all pairs of different computers from set H determines the binary routing matrix,

$$\mathbf{R} = (r_{ls})_{m_E \times m_P}, \quad (3)$$

where:

$$r_{ls} = \begin{cases} 1, & \text{if } e_l \in p_s, \\ 0, & \text{if } e_l \notin p_s, \end{cases}$$

and $e_l \in E$ for $l \in L$, and $p_s \in P$ for $s \in S$. Element r_{ls} of the matrix \mathbf{R} assumes a value of one if the link $e_l \in E$ belongs to the path $p_s \in P$, and a value of zero otherwise. Let $L_s = \{l \in L : r_{ls} = 1\}$ be a set of links belonging to the path $s \in S$. Let $S_l = \{s \in S : r_{ls} = 1\}$ be a set of paths passing through the link $l \in L$.

2.2 Model of the Message Passing Process

Let us assume that on every computer from set H computational processes are executed, called agents, which are exchanging messages. Let us also assume that every agent can pass messages to a single agent executed on another computer. Let $A_k = \{a_{ki}; i = 1, \dots, m_{A_k}\}$ be a set of agents executed on the computer h_k , $k \in K$, where $1 \leq m_{A_k} \leq m_H - 1$. Let

$A = \bigcup_{k \in K} A_k = \{a_1, \dots, a_{m_A}\}$ be a set of agents performed on all computers from set H , where:
 $m_A = \sum_{k \in K} m_{A_k}$, $m_H \leq m_A \leq m_H (m_H - 1)$.

Let us assume, that the messages are passed between selected pairs of different agents $(a_s, b_s) \in A \times A$, $a_s \neq b_s$, along the paths $p_s \in P$, for $s \in S$. In the pair (a_s, b_s) , the agent a_s is called the sender (or the source) and the agent b_s is called the receiver (or the sink) of messages passing through path p_s . The source a_s generates messages with transmission rates $y_s \geq 0$. After sending each packet, the source receives an ACK packet confirming receipt by the receiver of the previously sent packet. Let $u_s > 0$ be the length of time interval from the moment of generating the packet by the source a_s to the moment of receiving it by the ACK packet. Let $\hat{A} = \{\hat{a}_s \in A; s \in S\}$ be a set of selected agents from set A , where \hat{a}_s is an agent which sends the messages along path p_s .

The model of the data transfer along path p_s defines the ordered 4-tuple,

$$D_s = (T, U, Y, f_s), \quad s \in S, \quad (4)$$

where: $U = \mathbb{R}$, $Y = \mathbb{R}$, $f_s : U \times T \rightarrow Y$ is a map determining the transmission rate of the data generated by agent a_s , \mathbb{R} is a set of real numbers.

The model D , Eq. (1), defines the ordered 4-tuple,

$$D = (T, U, Y, \mathbf{f}),$$

where: $U = U^{m_A}$, $Y = Y^{m_A}$, $\mathbf{f} = (f_1, \dots, f_{m_A})^T$, f_s , Eq. (4), for $s \in S$.

3 The Multi-Objective Optimization Problem

The problem of network resource allocation of the distributed system M , Eq. (1), is to maximize the transmission rates $y_s \geq 0$ and $s \in S$, taking into account the limited capacities $c_l \in \mathbf{c}$ and $l \in L$ of all links, where \mathbf{c} , Eq. (2).

3.1 Formulation of the Multi-Objective Optimization Problem

The problem of network resource allocation can be formulated as follows,

$$\max_{\mathbf{y} \in Y_0} \left\{ \mathbf{q}(\mathbf{y}) = (q_1(\mathbf{y}), q_2(\mathbf{y}), \dots, q_{m_p}(\mathbf{y}))^T \right\}, \quad (5)$$

where:

- $Y_0 \subset Y$ is a set (or a region) of feasible solutions of the form,

$$Y_0 = \{ \mathbf{y} \in Y : \mathbf{g}(\mathbf{y}) \leq \mathbf{0}, \mathbf{y} \geq \mathbf{0} \}, \quad (6)$$

and $\mathbf{y} = (y_1, \dots, y_{m_p})^T$ is a vector of the message generation bitrates by agents from set \hat{A} , $\mathbf{g}: Y \rightarrow \mathbb{R}^{m_E}$ is a vector constraint function of the form $\mathbf{g}(\mathbf{y}) = \mathbf{R}\mathbf{y} - \mathbf{c}$, $\mathbf{g}(\mathbf{y}) = (g_1(\mathbf{y}), \dots, g_{m_E}(\mathbf{y}))^T$, and $g_l(\mathbf{y}) = \sum_{s \in S_l} y_s - c_l$, for $l \in L$;

- $\mathbf{q}: Y \rightarrow \mathbb{R}^{m_A}$ is a vector objective function (or a vector utility function) of agents from set \hat{A} , where $q_s: Y \rightarrow \mathbb{R}$ is an objective function of agent \hat{a}_s , for $s \in S$.

The values of the function q_s can be interpreted as the benefits of sending the data generated by agent $\hat{a}_s \in \hat{A}$ with bitrates $y_s \in Y$ through path $p_s \in P$. It is usually assumed that the functions q_s , $s \in S$, are honest in the specified sense [30]. As can be seen, set Y_0 , Eq. (6), is nonempty, closed, convex and compact.

A multi-dimensional set $Z = \{\mathbf{z} \in \mathbb{R}^{m_p} : \mathbf{z} = \mathbf{q}(\mathbf{y}), \mathbf{y} \in Y\}$ is called an *objective space*. A multi-dimensional set $Z_0 = \{\mathbf{z} \in \mathbb{R}^{m_p} : \mathbf{z} = \mathbf{q}(\mathbf{y}), \mathbf{y} \in Y_0\}$ is called an *objective region* or an *attainable region*. An element $\mathbf{z} = \mathbf{q}(\mathbf{y})$ is called an *objective vector*, where $\mathbf{z} = (z_1, \dots, z_{m_p})^T$, $z_s = q_s(\mathbf{y})$ is called an *objective scalar*, for $s \in S$. A vector $\hat{\mathbf{y}} \in Y_0$ is called a *feasible solution* or a *feasible decision*. An element $\hat{\mathbf{z}} = \mathbf{q}(\hat{\mathbf{y}})$ is called an *objective vector corresponding to the feasible solution $\hat{\mathbf{y}}$* , where: $\hat{\mathbf{z}} = (\hat{z}_1, \dots, \hat{z}_{m_p})^T$, $\hat{z}_s = q_s(\hat{\mathbf{y}})$ is called an *objective scalar corresponding to the feasible solution $\hat{\mathbf{y}}$* , for $s \in S$.

Let us assume that agent \hat{a}_s seeks to maximize the transmission rates y_s taking into account the limited capacities $c_l \in \mathbf{c}$ and $l \in L_s$ of all links lying on path $p_s \in P$. The problem of network resource allocation for agent \hat{a}_s consists in finding a vector $\hat{\mathbf{y}}_s^I \in Y_0$, such that,

$$q_s(\hat{\mathbf{y}}_s^I) = \max_{\mathbf{y} \in Y_0} q_s(\mathbf{y}), \quad s \in S, \quad (7)$$

where $\hat{\mathbf{y}}_s^I = (0, \dots, 0, \hat{y}_s^I, 0, \dots, 0)^T$ is an *ideal solution for agent a_s* , $\dim(\hat{\mathbf{y}}_s^I) = m_p$, for $s \in S$.

A vector $\hat{\mathbf{y}}^I = (\hat{y}_1^I, \dots, \hat{y}_{m_p}^I)^T$ is called an *ideal solution*, where $\hat{y}_s^I \in \hat{\mathbf{y}}_s^I$, for $s \in S$. A vector $\hat{\mathbf{z}}^I = (\hat{z}_1^I, \dots, \hat{z}_{m_p}^I)^T$ is called an *objective vector corresponding to the ideal solution $\hat{\mathbf{y}}^I$* , where: $\hat{z}_1^I \in \hat{\mathbf{z}}_1^I$, $\hat{\mathbf{z}}_s^I = (0, \dots, 0, \hat{z}_s^I, 0, \dots, 0)^T$, and $\hat{z}_s^I = q_s(\hat{\mathbf{y}}_s^I)$, for $s \in S$.

3.2 Solution of the Multi-Objective Optimization Problem

In multi-objective optimization, there does not typically exist a feasible solution that maximizes all objective functions simultaneously. Therefore, attention is paid to Pareto optimal solutions and weakly Pareto optimal solutions.

Definition 1. A feasible solution $\hat{\mathbf{y}} \in Y_0$ of the problem (5) is *Pareto optimal* (or *strict Pareto optimal*) if there does not exist another vector $\mathbf{y} \in Y_0$ such that $q_s(\mathbf{y}) \geq q_s(\hat{\mathbf{y}})$ for all $s \in S$ and $q_\nu(\mathbf{y}) > q_\nu(\hat{\mathbf{y}})$ for at least one index ν , $\nu \in S$. On the other hand, $\hat{\mathbf{y}} \in Y_0$ is *weakly Pareto optimal* if there does not exist another vector $\mathbf{y} \in Y_0$ such that $q_s(\mathbf{y}) > q_s(\hat{\mathbf{y}})$ for all $s \in S$.

There is a whole range of approaches to solving multi-objective optimization problems [10, 21, 22]. The most commonly used are approaches based on scalarizing multi-objective optimization problems [21, 22]. These include weighted sum methods and so-called no-preference methods.

Scalarizing multi-objective optimization problems

Scalarizing consists in the conversion of a multi-objective optimization problem into a single-objective optimization problem, such that optimal solutions to the single-objective optimization problem are weakly Pareto optimal solutions to the multi-objective optimization problem. A general formulation for the scalarization of multi-objective optimization is thus,

$$\max_{\mathbf{y} \in Y_0} f(\mathbf{q}(\mathbf{y}), \boldsymbol{\theta}), \quad (8)$$

where: $\boldsymbol{\theta} \in \mathbb{R}^{m_p}$ is a vector of parameters, $f: \mathbb{R}^{m_p} \times \mathbb{R}^{m_p} \rightarrow \mathbb{R}$ is an objective function.

The following theorem define the necessary and sufficient conditions for an optimal solution of the problem (8) to be weakly Pareto optimal and Pareto optimal.

Theorem 1.

1. If a function f is strictly increasing and if $\hat{\mathbf{y}} \in Y_0$ is an optimal solution of the problem (8), then $\hat{\mathbf{y}}$ is weakly Pareto optimal.
2. If a function f is strictly increasing and if the solution $\hat{\mathbf{y}} \in Y_0$ of the problem (8) is unique, then $\hat{\mathbf{y}}$ is Pareto optimal.

Proof. Proof can be found in [39].

Weighted sum methods

Weighted sum methods require additional information about the decision-maker's preferences. The most popular is the so-called linear scalarization in which the problem (8) is converted to the following form,

$$\max_{\mathbf{y} \in Y_0} \{f^w(\mathbf{q}(\mathbf{y}), \mathbf{w}) = \mathbf{w}^T \mathbf{q}(\mathbf{y})\}, \quad (9)$$

where: $\mathbf{w} = (w_1, \dots, w_{m_p})^T$ is a vector of weights of the objective functions such that $\sum_{s \in S} w_s = 1$, and $w_s > 0$, for $s \in S$, $f^W : \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}$ is an objective function which is called a *weight function*.

Corollary 1. Any feasible solution $\hat{\mathbf{y}}^W \in Y_0$ of the problem (9) is weakly Pareto optimal.

Proof. The proof follows directly from Theorem 1, since the function f^W , Eq. (9), is fulfilling an assumption determined in point 1 of this theorem.

No-preference methods

These methods do not require any additional information about the decision-maker's preferences. The most popular are the distance method and so-called Chebyshev type methods.

The distance method consists in converting the problem (8) to the following form,

$$\min_{\mathbf{y} \in Y_0} \left\{ f^D(\mathbf{q}(\mathbf{y}), \hat{\mathbf{z}}^I) = \|\mathbf{q}(\mathbf{y}) - \hat{\mathbf{z}}^I\|_p \right\}, \quad (10)$$

where: $\hat{\mathbf{z}}^I$ (point 3.1), $f^D : \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}$ is an objective function which is called a distance function, $\|\cdot\|_p$ is any L_p norm.

Corollary 2. Any feasible solution $\hat{\mathbf{y}}^D \in Y_0$ of the problem (10) is weakly Pareto optimal.

Proof. The proof is analogous to the proof of corollary 1.

Among so-called Chebyshev type methods [10, 21, 22] the most well-known is the method which converges the problem (8) to the following form,

$$\min_{\mathbf{y} \in Y_0} \left[f^A(\mathbf{q}(\mathbf{y}), \boldsymbol{\lambda}) = \rho \boldsymbol{\lambda}^T (\mathbf{q}(\mathbf{y}) - \hat{\mathbf{z}}^I) + \max_{s \in S} \left\{ \lambda_s (\mathbf{q}_s(\mathbf{y}) - \hat{z}_s^I) \right\} \right], \quad (11)$$

where: $\rho > 0$ is some sufficiently small scalar [22], $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_{m_p})^T$ is a vector of non-negative coefficient used for scaling purposes, that is, for normalizing objective functions of different magnitudes, $f^A : \mathbb{R}^{m_p} \times \mathbb{R}^{m_p} \rightarrow \mathbb{R}$ is an objective function. Typically, these coefficients take the following values,

$$\lambda_s = \begin{cases} 1, & \text{if } \hat{z}_s^I = 0, \\ \frac{1}{\hat{z}_s^I}, & \text{if } \hat{z}_s^I \neq 0, \end{cases} \quad s \in S.$$

Corollary 3. Any feasible solution $\hat{\mathbf{y}}^A \in Y_0$ of the problem (11) is weakly Pareto optimal.

Proof. The proof is analogous to the proof of corollary 1.

Example 1. Let us consider the issue of multi-objective optimization of network resource allocation in a simple distributed system, whose structure represents a net $N = (V, E, \mathbf{c})$ (Fig. 1), where: $V = H \cup R$, $H = \{h_1, \dots, h_{m_H}\}$, $m_H = 3$, is a set of computers, $R = \{r\}$ is a set of network routers, $E = \{e_1, \dots, e_{m_E}\}$, $m_E = 3$, is a set of one-way network links, $\mathbf{c} = (c_1, \dots, c_{m_E})^T$ is a vector of capacities of all network links.

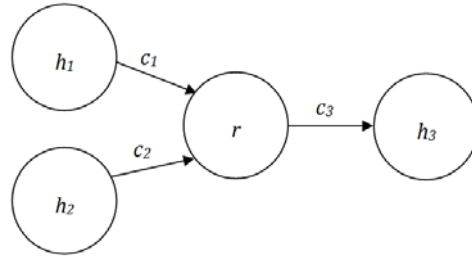


Figure 1. Model of the distributed system structure

The sets K , S and L take the following forms: $K = \{1, 2, 3\}$, $S = \{1, 2\}$, $L = \{1, 2, 3\}$. Set P takes the following form $P = \{p_1, \dots, p_{m_P}\}$, $m_P = 2$, where: $p_1 = (h_1, r, h_3)$, $p_2 = (h_2, r, h_3)$. The sets A_k and $k \in K$, take the following forms: $A_1 = \{a_{11}\}$, $A_2 = \{a_{21}\}$, $A_3 = \{a_{31}, a_{32}\}$. Set A takes the form $A = \bigcup_{k \in K} A_k = \{a_{11}, a_{21}, a_{31}, a_{32}\} = \{a_1, \dots, a_{m_A}\}$, $m_A = 4$. Let us assume, that the messages are passed between pairs of agents: $(a_1, b_1) \in A \times A$ and $(a_2, b_2) \in A \times A$, along the paths, respectively, $p_1 \in P$ and $p_2 \in P$, where: $b_1 = a_3$ and $b_2 = a_4$. Set \hat{A} takes the form $\hat{A} = \{a_1, a_2\}$.

The matrix \mathbf{R} , Eq. (3), takes the form,

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

Let us assume that vector \mathbf{c} has the form $\mathbf{c} = (100, \dots, 100)^T$, which means that all the links have the same capacity, i.e. $c_l = 100$ [Mbps], for $l \in L$.

Let us also assume that all elements from sets H and R are passing data using the TCP protocol [30], for which the utility functions q_s and $s \in S$, assume the following forms,

$$q_s(\mathbf{y}) = -\frac{1}{u_s y_s}, \quad s \in S,$$

where: $u_s > 0$ is the time interval length from the moment of generating a packet by the source a_s to the moment of receiving an ACK packet by this source, $y_s > 0$ is the transmission rate of the source a_s . Let us assume that $u_1 = 0.1$ [s] and $u_2 = 0.2$ [s].

To solve the optimization problems (7) the interior point method [24] has been applied.

Table 1 shows the results of solving the problem (7), where: $\hat{\mathbf{y}}^l = (\hat{y}_1^l, \hat{y}_2^l)^T$, $\hat{\mathbf{z}}^l = \mathbf{q}(\hat{\mathbf{y}}^l)$, g_l , Eq. (6), for $l \in L$. This table shows that the constraint $g_3(\hat{\mathbf{y}}^l) \leq 0$ is not satisfied.

Table 1. The results of solving the problem (7)

$\hat{\mathbf{y}}^l$	$\hat{\mathbf{z}}^l$	$g_1(\hat{\mathbf{y}}^l)$	$g_2(\hat{\mathbf{y}}^l)$	$g_3(\hat{\mathbf{y}}^l)$
$(100, 100)^T$	$(-0.1, -0.05)^T$	0	0	10 0

Let us assume that we are searching for single solutions to problems (9)-(11).

Table 2 shows the results of solving the problem (9), wherein it is assumed that a vector of weights takes the value of $\mathbf{w} = (0.75, 0.25)^T$, where: $\hat{\mathbf{y}}^w = (\hat{y}_1^w, \hat{y}_2^w)^T$, $\hat{\mathbf{z}}^w = \mathbf{q}(\hat{\mathbf{y}}^w)$. This table shows that all the constraints are satisfied.

Table 2. The results of solving the problem (9)

$\hat{\mathbf{y}}^w$	$f^w(\hat{\mathbf{y}}^w)$	$\hat{\mathbf{z}}^w$	$g_1(\hat{\mathbf{y}}^w)$	$g_2(\hat{\mathbf{y}}^w)$	$g_3(\hat{\mathbf{y}}^w)$
$(71.0101, 28.9898)^T$	- 0.148737	$(-0.140825, -0.172474)^T$	- 28.9899	- 71.0102	- 0.0000611117

Table 3 shows the results of solving the problem (10), where: $\hat{\mathbf{y}}^D = (\hat{y}_1^D, \hat{y}_2^D)^T$, $\hat{\mathbf{z}}^D = \mathbf{q}(\hat{\mathbf{y}}^D)$. This table shows that all the constraints are satisfied.

Table 3. The results of solving the problem (10)

$\hat{\mathbf{y}}^D$	$f^D(\hat{\mathbf{y}}^D)$	$\hat{\mathbf{z}}^D$	$g_1(\hat{\mathbf{y}}^D)$	$g_2(\hat{\mathbf{y}}^D)$	$g_3(\hat{\mathbf{y}}^D)$
$(58.5787, 41.4213)^T$	0.1	$(-0.170711, -0.120711)^T$	- 41.4213	- 58.5787	- 0.0000406612

Table 4 shows the results of solving the problem (11), wherein it is assumed that a parameter ρ takes the value of $\rho = 0.1$, where: $\hat{\mathbf{y}}^A = (\hat{y}_1^A, \hat{y}_2^A)^T$, $\hat{\mathbf{z}}^A = \mathbf{q}(\hat{\mathbf{y}}^A)$. This table shows that all the constraints are satisfied.

Table 4. The results of solving the problem (11)

$\hat{\mathbf{y}}^A$	$f^A(\hat{\mathbf{y}}^A)$	$\hat{\mathbf{z}}^A$	$g_1(\hat{\mathbf{y}}^A)$	$g_2(\hat{\mathbf{y}}^A)$	$g_3(\hat{\mathbf{y}}^A)$
$(50.0, 50.0)^T$	1.4	$(-0.2, -0.1)^T$	-	-	0

Figure 2 shows the set of feasible solutions Y_0 , Eq. (6), with marked points: $\hat{\mathbf{y}}^I$ (table 1), $\hat{\mathbf{y}}^W$ (table 2), $\hat{\mathbf{y}}^D$ (table 3) and $\hat{\mathbf{y}}^A$ (table 4).

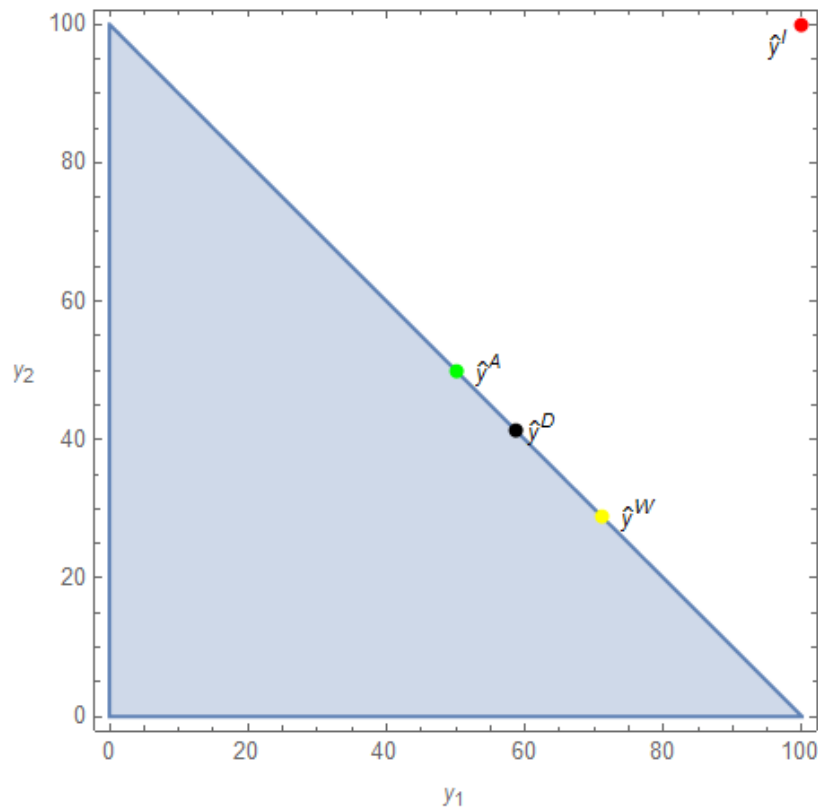


Figure 2. A plot of set Y_0 with marked points: $\hat{\mathbf{y}}^I$ (red dot), $\hat{\mathbf{y}}^W$ (yellow dot), $\hat{\mathbf{y}}^D$ (black dot), $\hat{\mathbf{y}}^A$ (green dot)

Figure 3 shows a fragment of the set Z_0 with marked points: $\hat{\mathbf{z}}^I$ (table 1), $\hat{\mathbf{z}}^W$ (table 2), $\hat{\mathbf{z}}^D$ (table 3), and $\hat{\mathbf{z}}^A$ (table 4).

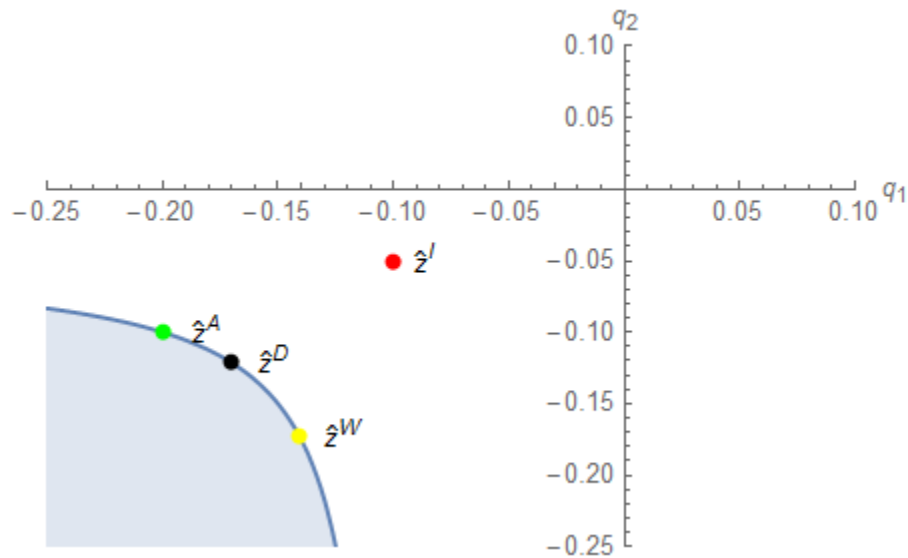


Figure 3. A plot of a fragment of the set Z_0 with marked points: \hat{z}^I (red dot), \hat{z}^W (yellow dot), \hat{z}^D (black dot), \hat{z}^A (green dot)

It is worth noting that Theorem 1 demonstrates that solutions \hat{y}^W , \hat{y}^D and \hat{y}^A are weakly Pareto optimal.

□

Summary

The paper presents the use of multi-objective optimization methods to solve the problem of network resource allocation in distributed systems. An important feature of the proposed approach is that it allows to take into account various criteria for solving the considered problem. To solve multi-criteria optimization problem some scalarizing functions have been used. It is worth noting that the use of these functions guarantees obtaining weakly Pareto optimal solutions.

Follow-up works should consist in applying both exact and approximation methods for solving the problem defined by the formula (8). Many studies [5, 10, 32, 38] indicate that promising approaches for solving the considered problem use evolutionary algorithms and tabu search. Apart from the fact that they often give close-to-optimal solutions, it is possible to implement them simultaneously [16, 33, 34]. Due to the fact that these methods can be performed in a partially asynchronous way, it is possible to try to use them for effectively solving many substantial problems of computer science, including the problem of decentralized control with communication between controllers [26].

References

1. Bolch G., Greiner S., de Meer H., Trivedi K. S., (2006). *Queueing Networks and Markov Chains: Modeling and Performance Evaluation with Computer Science Applications*, Wiley-Blackwell.
2. Boxma O., Koole G., Liu Z., (1994). *Queueing-theoretic Solution Methods for Models of Parallel and Distributed Systems*, in *Performance Evaluation of Parallel and Distributed Systems Solution Methods*, CWI Tract 105 and 106.
3. Calafiore G. C., (2009). *Distributed Randomized Algorithms for Probabilistic Performance Analysis*, *Systems & Control Letters*, Volume 58, Issue 3, pp. 202-212.
4. Casale G., (2011). *Exact Analysis of Performance Models by the Method of Moments*, *Performance Evaluation*, Volume 68, Issue 6, pp. 487-506.
5. Coello Coello C.A., Lamont G.B., van Veldhuizen D.A. (2007). *Evolutionary Algorithms for Solving Multi-Objective Problems: Genetic Algorithms and Evolutionary Computation*, Springer.
6. Committee on Network Science for Future Army Applications (2005). *Network Science*, National Academies Press.
7. Cowie J., Nicol D., Ogielski A., (1999). *Modeling the Global Internet*, *Computing in Science & Engineering*, Vol. 1, No. 1, pp. 42–50.
8. Coulouris G., Dollimore J., Kindberg T., Blair G. (2011). *Distributed Systems: Concepts and Design*, Addison-Wesley.
9. Deane J.H.B., Jefferies D.J., Smythe C., (1993). *Chaotic Traffic Flow in Local Area Networks*, *Proceedings of the 11th European Conference on Circuit Theory and Design*, Davos, Switzerland, pp. 843-848.
10. Deb K., (2001). *Multi-Objective Optimization using Evolutionary Algorithms*, John Wiley & Sons.
11. Diallo O., Rodrigues J. J. P. C. , Sene M., *Performances Evaluation and Petri Nets*. In: *Modeling and Simulation of Computer Networks and Systems: Methodologies and Applications*, Obaidat M. S., Zarai F., Nicopolitidis P. (Eds.), Morgan Kaufmann Publishers.
12. Dirx E., Verboven F., Tiberghien J., (1990). *Distributed Simulation of Computer Networks*, *Microprocessing and Microprogramming*, Volume 30, Issues 1–5, pp. 215-220.
13. Donoso Y., Fabregat R., (2007) . *Multi-Objective Optimization in Computer Networks Using Metaheuristics*. Auerbach Publications Taylor & Francis Group.
14. Harchol-Balter M., (2013). *Performance Modeling and Design of Computer Systems: Queueing Theory in Action*, Cambridge University Press.

15. Hasegnum M., Mlii G., Mixno A., (2001). Applications of Nonlinear Prediction Methods to the Internet Traffic, The 2001 IEEE International Symposium on Circuits and Systems, ISCAS 2001, Vol. 2, pp. 169 - 172.
16. Jaeggi D., Asselin-Miller Ch., Parks G., Kipouros T., Bell T., Clarkson J., (2004). Multi-objective Parallel Tabu Search. In: Yao X., Burke E., Lozano J.A., Smith J., Merelo-Guervós J.J., Bullinaria J.A., Rowe J., Tino P., Kabán A., Schwefel H.P. (Eds.). Parallel Problem Solving from Nature - PPSN VIII. Proceedings of the 8th International Conference, Birmingham, UK, September 18-22, 2004, pp. 732-741.
17. Jean-Marie A., Lefebvre-Barbaroux S., Liu Z., (1998). An Analytical Approach to the Performance Evaluation of Master-Slave Computational Models, Parallel Computing, Volume 24, Issues 5-6, pp. 841-862.
18. Lewis T. G., 2009. Network Science: Theory and Applications, John Wiley & Sons.
19. Lin Y.K., , Yeh Ch. T. (2012). Multi-objective optimization for stochastic computer networks using NSGA-II and TOPSIS. European Journal of Operational Research, Volume 218, Issue 3, 2012, pp. 735-746.
20. Low H., Paganini O., Doyle J. C., (2002). Internet Congestion Control, IEEE Control Systems Magazine, pp. 28-43.
21. Marler T. (2009). Multi-Objective Optimization. Concepts and Methods for Engineering, VDM Verlag.
22. Miettinen K., Mäkelä, M. M., (2002). On Scalarizing Functions in Multi-Objective Optimization, OR Spectrum, May 2002, Volume 24, Issue 2, pp 193-213.
23. Nikolaidou M., Anagnostopoulos D., (2003). A Distributed System Simulation Modelling Approach, Simulation Modelling Practice and Theory, Volume 11, Issues 3-4, 15 July 2003, pp. 251-267.
24. Nocedal J., Wright S., (1999). Numerical Optimization. Springer.
25. Paxson V., Floyd S., (2001). Difficulties in Simulating the Internet, IEEE/ACM Transactions on Networking, Vol. 9, No 4, pp. 392-403.
26. Schuppen van Jan H., (2009). Decentralized Control with Communication Between Controllers. In: Blondel V. D., Megretski A. (Eds.). Unsolved Problems in Mathematical Systems and Control Theory. Princeton University Press, 2009.
27. Shamma J. (Ed.), (2007), Cooperative Control of Distributed Multi-Agent Systems, Wiley-Blackwell.
28. Shestak V., Chong E. K. P., Maciejewski A. A., Siegel H. J., (2012). Probabilistic Resource Allocation in Heterogeneous Distributed Systems with Random Failures, Journal of Parallel and Distributed Computing, Volume 72, Issue 10, October 2012, pp. 1186-1194.

29. Snyder H., Kurtze D., (1996). Chaotic Behavior in Computer Mediated Network, *Information Processing & Management*, Vol. 32, No. 5, pp. 555-562.
30. Srikant R., (2004). *The Mathematics of Internet Congestion Control*, Birkhäuser, Berlin.
31. Takayasu M., Fukuda K., Takayasu H., (1999). Application of Statistical Physics to the Internet Traffics, *Physica A* 274, pp.140-148.
32. Talbi E.G., Basseurc M., Nebrod A.J., Albad E., (2012). Multi-objective Optimization using Metaheuristics: Non-standard Algorithms. *International Transactions in Operational Research*, 19 (2012), pp. 283–305.
33. Talbi E.G., Mostaghim S., Okabe T., Ishibuchi H., Rudolph G., Coello Coello C.A., (2008). Parallel Approaches for Multi-objective Optimization. In: Branke J., Deb K., Miettinen K., Slowinski R. (Eds.), *Multiobjective Optimization. Interactive and Evolutionary Approaches*, Springer. *Lecture Notes in Computer Science*, Vol. 5252, 2008, pp. 349–372.
34. Toro Negro de F., Ortega J., Ros E., Mota S., Paechter B., Martin J. M., (2004). PSFGA: Parallel Processing and Evolutionary Computation for Multiobjective Optimisation. *Parallel Computing*, Elsevier, 30 (5–6), May 2004, pp. 721–739.
35. Trivedi K. S., Vaidyanathan K., Selvamuthu D., (2015). Markov Chain Models and Applications. In: *Modeling and Simulation of Computer Networks and Systems: Methodologies and Applications*, Obaidat M. S., Zarai F., Nicopolitidis P. (Eds.), Morgan Kaufmann Publishers.
36. Tseng P., Bertsekas D. P. , Tsitsiklis J. N., (1990). Partially Asynchronous, Parallel Algorithms for Network Flow and Other Problems, *SIAM Journal on Control and Optimization*, Vol 28, No. 3, May 1990, pp. 678-710.
37. Vattay G., Diriczi K., Fekete A., Kocarev L., Marodi M., Steger J., (2010). Statistical Properties of Chaos in Communication Networks. In: *Complex Dynamics in Communications Networks* , Kocarev L., Vattay G. (Eds.), Springer.
38. Veldhuizen van D.A., Lamont G. B., (2000). Multiobjective Evolutionary Algorithms: Analyzing the State-of-the-Art. *Evolutionary Computation*, Summer 2000, Vol. 8, No. 2, pp. 125-147.
39. Wierzbicki A.P., (1986). A Methodological Approach to Comparing Parametric Characterizations of Efficient Solutions. In: *Large-Scale Modelling and Interactive Decision Analysis*, Fandel G., Grauer M., Kurzhanski A. (Eds.), *Lecture Notes in Economics and Mathematical Systems*, Volume 273, pp. 27-45.
40. Willinger W., Paxson V., (1998). Where Mathematics Meets the Internet, *Notices of the AMS*, Volume 45, Number 8, pp. 961-970.
41. Xhafa F., Abraham A. (Eds.), (2008). *Metaheuristics for Scheduling in Distributed Computing Environments*. Springer-Verlag.