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An approach of explaining math function graphs through the sound representation for blind students

Abstract. One of the key abilities that should be learned by every student is mastering math skills. It is particularly difficult for blind students who have problems with access to information that is usually presented graphically for common pupils. In mathematics one such information is the presentation of graphs. However there are several drawing techniques for the blind, but they are expensive, and hard for common using. One way to solve this problem may be using a special sound representation of math graphics for the blind students. In this article, an approach allowing to provide making such presentation possible is presented. Briefly it is grounded on the conversion of graphic data of the math drawing to sound wave with proper assets.

Keywords. Blind children, Math graphs, Sound representation, Teaching mathematics

1. Introduction

One of the key issues that should be learned by every student is mastering math skills. It is particularly difficult for the blind who have problems with access to information that is usually presented graphically for the sighted. In mathematics one such information is the structural presentation of the formulas and equations and graphical presentation of graphs. A problem of adequate presentation of mathematic formulas for the blind was partially solved by using Braille mathematical notation or reading formulas by speech synthesizer [3]. But much more difficulties the blind faces in the case of using math graphs. However, there are a few drawing techniques for the blind [6], but they are expensive and hard for common using.

One way to solve this problem may be using a specially developed sound representation of math graphics for the blind student. Such solution will be described briefly in this article.

2. The related works

Research under methods of creating tactile graphic for the blind has been conducted in a long time. The most traditional way of making tactile graphic for the blind was the technique of gluing of sticks and bits of cardboard on the paper sheet. This technique had been used before modern computer solutions appeared and it is sometimes still used to create simple graphics and models for blind children. In teaching of geometry, wooden or plastic models of the plane figures or 3-dimensional figures as well as skeletons made of plastic rods are also used. One of the classical solution is using the special drawing board for the blind [10]. It is a drawing board covered with a layer of rubber, on which a plastic film is placed. Due to the fact that the direction of the sharp instrument on the surface of such a film leads to drooping of the rubber itself, the production of the convex line on the film is possible. This technique allows the blind to perform the geometric constructions, although their accuracy is not great (about 3 mm). Another option, which can be used to create drawings for the blind is use of brailler [9]. It is a special typewriter, which allows to emboss points on a paper that form the characters in the Braille alphabet. A disadvantage of this solution is the complexity of creating diagonal lines in the figure and the complete lack of creating arches. A second disadvantage of this method is forcing the creation of the drawing from top to bottom, because in order to return to the area already embossed, inserting the card into the rollers of the brailler is necessary. It causes crushing and rubbing previously drawn points.

Another example of technique that can facilitate the creation of drawings for the blind are Braille embossers [9]. Embossers are special printers that are suitable for printing characters in the braille system. The best quality of printed figures can be achieved by embossers of ViewPlus series [11] because they can print dots about 5 different diameters and higher

resolution (about 70 dpi). The worst quality of the picture is reached of popular printers of Index series [4], because they print dots of only one diameter. The main disadvantage of this solution is the price of Braille embossers, which varies from 8 to several dozens of thousands of dollars.

A slightly different approach is the swell paper method [1]. This is the type of paper in which the drawing using a standard laser printer can be placed. When the sheet of paper is heated in a special machine, a layer that covers paper becomes convex in places where items are drawn. The height of Swell depends on the thickness of the line, a thick stroke makes the paper more convex than drawing thin lines. The color of the drawn lines has also its meaning, the darker line becomes more convex than lighted one. The figure produced in this way can be recognized with touching by blind student with his fingers. Only disadvantage of this solution is the high price of heating machine and high price of swell paper sheet.

Another example of the distribution of tactile graphic is a technique that uses so-called brailon [7]. This technique is to create a model of the object (called the matrix) of a material resistant to high temperature. Then this model is covered with a plastic film and heated in a vacuum using special device. In this way, the top-side shape of the model is copied into convex shape on the plastic film. This method allows to represent the differentiation of the surface of the object and selection of model layers. This method is primarily used for creating educational drawings, i.e. anatomy, because using Brailon makes easy to imagine the breakdown of tissues, muscles, organs. This method is widely used in schools for the blind. The disadvantage of this solution is also the relatively high cost of machine (presses for vulcanization of the drawings).

The approaches with representing of graphics by sound for blind people are not almost a new idea. One of the most known implementation is the vOICe vision technology [12] which offers the experience of live camera views through image-to-sound renderings. In this solution the user is equipped with special software, glasses with camera and headset. Due to them he can recognize his environment as a properly prepared audio signals that is generated from image transferred from the camera. There were also attempts to use this technology with electrodes placed in retinal and cortical implants. Another example may be an audio program called Audio Graphing Calculator [3]. In addition to the analysis of simple mathematical functions, the user has access to more advanced options such as possibilities to calculate integrals and derivatives. After selecting the view of function equation, the program displays basic information about functions (data collection, median standard deviation), which can then be read by the reader. After the appearance of the math graph the user has the ability to play it with the sound. An additional available option is the possibility to play of sound for

most important points of the function graph. However it is good solution, its disadvantage for polish young students is the lack of support of the Polish language.

Another example of a software that has the functionality close to the application proposed in this work is a MathTracks system [8]. It allows blind people to explore and study graphs of mathematical functions. It consists of 4 components. The first two of them are: BlindMath and TalkingMath. They are designed for writing and reading mathematical functions. BlindMath allows to enter functions and creates a file in LaTeX format, which contains information about the graph of the function. TalkingMath is responsible for reading the mathematical functions, no matter how complex they are. The other two components allows for multimodal exploration of graphs. A BlindGraph module uses previously obtained LaTeX file to display function graph on a specially prepared screen. The last component called AudioTact is a sensor, which is mounted on the pen or, in the case of a touch screen, on the finger of the user. Due to this, when user touches the screen and input software graph, the program produces the left channel audio signal and right channel vibrations that are transferred to the AudioTact device, than they can be perceived by the user.

Another solution is a PlatMat system [2], that has been implemented in the Institute of Mathematical Machines in Warsaw. In the PlatMat application a student can explore a function graph by sliding his finger on the touch screen where drawing is presented. When the student encounters a line, he also hears the continuous sound tone. Similarly, when he meets on the axis of the coordinate system, he can hear the sound at a different frequency. In addition, when the student performs a tap gesture on the line, he can hear its brief description read by speech synthesizer. The PlatMat system was in some sense a continuation of solution presented in this article.

3. Representing math graphs using sound effects

It seems a good idea to solve the problem of explaining math graphs for the blind would be use a stereo sound with appropriate pitch, tone and volume. A simple intuitive approach could be the identification of the pitch of the sound with the y axis coordinates of the point on the graph and the x-coordinate of point with the time and position of the sound in the panorama between stereo channels. This method makes an easy way to explain two-dimensional diagrams for the blind student. It can be also extended to present 3D graphics by changing another sound parameters such as volume or tone, or to use so-called binaural sounds and the holophony phenomena [5]. The basic version of this method mentioned above was implemented in the form of the sample application.

3.1. How it works

Let us to imagine a situation, when a blind pupil attends lessons in the class in ordinary public school. The teacher of mathematics has to perform a lesson on quadratic functions in this class. He needs to find a way to show the function graphs for his blind student. Unfortunately, a teacher does not have access to Braille printer or any other technology allowing to make tactile drawings. However, he has access to our software that allows showing function's graphs in the form of stereo sound signals. Before the lesson, the teacher is launching the program and uses this to prepare graphs of quadratic functions he needs for his lesson. The program offers three options: drawing the graph by entering the linear or quadratic equation, drawing the graph of trigonometric function as well as drawing a function graph through the provision of a set of data points. In this case, the teacher is selecting the option to create a quadratic function of equation $y = 2x^2$.

The mentioned function graph is presented in figure 1.

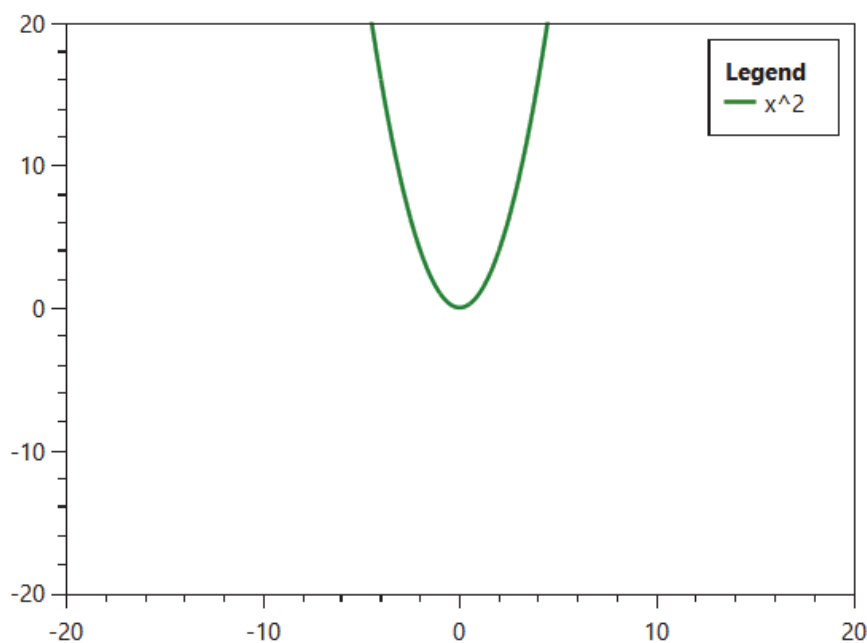


Figure 1. A graph of quadratic function

Moreover, a teacher is creating a few other graphs of quadratic functions. During the lesson, when the teacher wants to show the function graph for his sighted students, he is drawing it on the blackboard and is making a discussion of its appearance and properties. In this moment, he also is asking his blind student to launch application that allows presenting graphs using sound signals. A student is loading a chart prepared by teacher into his program and is running a command "play chart by sound". At this point, a student is hearing a sound that represents a quadratic function. The sound is playing in the stereo panorama, therefore,

first it is heard in the left channel, and then it is moving towards the right channel. Let us notice that this movement reflects the visual cursor movement along the x-axis that is occurred from left to right. In addition, the frequency of the sound that is initially high, is very much reduced to a certain point, from which again it begins to raise. This shows the value of a quadratic function which can be read on the x-axis by sighted student. Moreover, at the time when the signal exceeds the cursor in x or y axis, additional short sound signals are heard.

Let us assume, that in such way, a blind student can build the picture of a function in his mind. It replaces the ability to view a graph by his sighted colleagues who can see it on the board.

3.2. The concept of sound representation of math graphs

As it is commonly known, mathematical functions graphs are presented in the computer applications in the form of graphic data, i.e. set of pixels in raster graphics, or set of objects and their parameters in vector graphics. Users with special needs such as the blind do not have full access to this kind of information because they perceive their surrounding mainly from the perspective of sounds that reach them. So this graphic information given to a blind may be used in different kind of sounds.

The concept of presenting graphics to the blind in the form of sound is based on a simple observation that points have x and y coordinates stored as pairs of numbers. If we take stereo sound of changing frequency we can notice, that we have two variable parameters, namely, the position of the sound in the stereo panorama, and frequency, i.e. the pitch of the sound. The simple conclusion arises, therefore the transformation of the Cartesian coordinates of a point on these sound characteristics is very easy. We have to also take into account that graphic point is observed always as a snapshot in a well-defined moment unlike the sound should be played after a certain duration time. This fact can also be used to represent the x-coordinate of each point of the graph in such a way that the sounds that represent points of smaller x coordinate will be played earlier than sounds that represents points with greater x coordinate. So what is the method for the submission of math graphs in the form of sound? Let us start with the input data that is a list of pairs. Each represents coordinates of a particular point in the function graph. Then this list is sorted on the first element of each pair, which represent x coordinates of each point. Using this input data, a table containing values that represent frequency changes of sound to be played is generated.

So for each y_i and y_{i+1} a frequency change is calculated

$$f_i = (y_i + 1) * 5.$$

Each frequency change is then added to frequency of each sound sample frequency. Thus, the changes of y coordinate of function graph points will be represented by adequate changes of the frequency of the reproduced sound. Then an output table consists of wave representation of each sound sample can be generated. At first initial sample rate s is set to value of 44100. Then for each sample a table of sound wave values is calculated. For each partial sample rate s_i of range from 0 to s wave value

$$v_i = \sin\left(\frac{2\pi s_i}{s}\right) * f_i.$$

Also other necessary parameters of each sound sample object is set is such as volumes of left and right stereo channel and portamento time. This property is the time required for a smooth transition between the two nearby played sound samples. Volumes of left and right channel also depend on x coordinate of each point from input list. Initially, when x coordinates are less than 0 a left channel volume is high and right channel volume is low. When x axis grows up then the right channel volume becomes higher and left channel volume becomes lower. In this way the user can experience the effect that sound moves from the left side of stereo panorama to the right. Then, after fixing all these properties, final signal is transmitted.

4. Conclusions

In this article, an approach to the presentation of graphs of mathematical functions for the blind children was described. It uses the sounds with appropriate parameters to create a mental view of function graph in the imagination of blind pupil.

The idea was implemented as a simple desktop application at a frame of engineering dissertation of Jakub Mańkowski under the supervision of Dariusz Mikulowski, PhD at the Institute of Computer Science, at University of Natural Sciences and Humanities in Siedlce. The software has been tested for 2 sighted and 2 blind users. They created a few dozen graphs, of quadratic, trigonometric and linear functions, and then listened to their sound performances. The idea was met with positive feedback and users have suggested only few fixes on usability of user interface.

As a conclusion we can say that idea may be successfully progressed and extended. For example, in addition to frequency changes and distribution of the sound in the stereo

panorama that was used in the prototype software, may be the use of such audio parameters like modulation and volume, and the rest of the waveform can be applied.

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